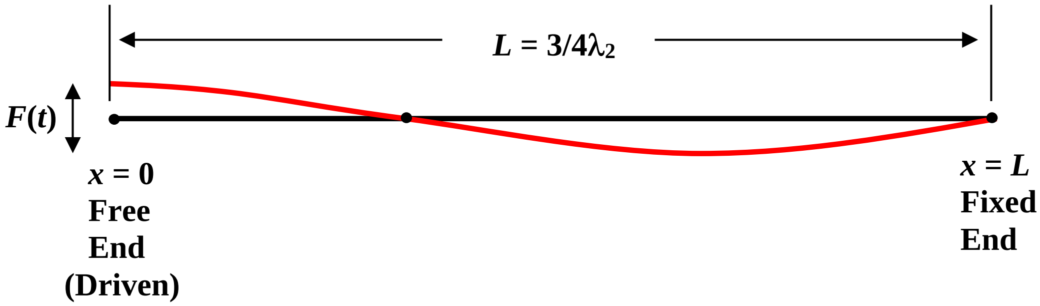
vibration in this situation will be different than those associated with fixed end-fixed end boundary conditions. For free end-fixed end boundary conditions, we must have a displacement *anti-node* at the x = 0 free end (because the slope of the string at x = 0, $\partial y(x=0, t)/\partial x = 0$). At the x = L fixed end, we must have a displacement *node* y(x=0, t) = 0. In the figure below, we show the second lowest mode of vibration of a standing wave on a string with free end-fixed end boundary conditions.



The transverse standing wave modes of vibration on a string with free end-fixed end boundary conditions are such that $L = {}^{1}/_{4}\lambda_1, {}^{3}/_{4}\lambda_2, {}^{5}/_{4}\lambda_3, {}^{7}/_{4}\lambda_4, {}^{9}/_{4}\lambda_5$, ... i.e. $L = {}^{(2n-1)}/_{4}\lambda_n$, or wavelength, $\lambda_n = {}^{4}/_{(2n-1)}L$, and wavenumber, $k_n = 2\pi/\lambda_n = {}^{(2n-1)\pi}/_{2L}$ and frequency, $f_n = v_x/\lambda_n = {}^{(2n-1)v_x}/_{4L}$, with *n* = 1, 2, 3, 4, ...

For driving frequencies, $f = f_n$ associated with the production of transverse standing wave modes of vibration on a string (i.e. *resonances*) with free end-fixed end boundary conditions, the complex mechanical input impedance, Z_n^{input} of the driven string at the driving point, x = 0 for the nth mode of this type of transverse vibration is given by:

$$Z_n^{input} \equiv \frac{F(t)}{u_{ny}(x=0,t)} = \frac{k_n T\cos(k_n L)}{i\omega_n \sin(k_n L)}$$

$$= -i\frac{k_n T}{\omega_n}\cot(k_n L) = -i\frac{T}{v_x}\cot(k_n L) = -i\frac{T}{v_x}\cot\left(\tfrac{(2n-1)\pi}{2}\right) = -iZ_o\cot\left(\tfrac{(2n-1)\pi}{2}\right) = 0$$

because $\cot[(2n-1)\pi/2] = 0$ for n = 1, 2, 3, 4, ... Thus, at these resonant frequencies, the complex driving force, F(t) can very easily transfer energy from the transversely vibrating free end support to the string, generating a standing wave on the string at this same frequency. (This same energy will come back out of the string a little while later.)

One can also run this same process *backwards* in time (i.e. use the symmetry principle of *time-reversal invariance*): Conversely, if one plucks a string such that e.g. its fundamental, or one of the higher harmonics of the string vibrates at one of these resonant frequencies, the vibrating forces acting on this “free” end support will efficiently transfer energy from the string to the “free” end support at that vibrational frequency, causing it to vibrate, thereby draining/damping energy from the vibrating string. In other words, for a guitar with such a “free” end support, the *sustain* of this guitar will be very significantly degraded if the string has a vibrational mode at (or near) such a resonant frequency of a free end-fixed end support system - the “free” end being an approximation to the bridge on an acoustic (or hollow-body electric) guitar, or an approximation to the nut at the end of the neck on a solid body electric guitar, or e.g. a *massless* slide/bottleneck!