Thus, the complex transverse velocity, $u_y(x, t) = \partial y(x, t)/\partial t$ of the driven string, at an arbitrary point, x and time, t is given by:

$$u_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \frac{i\omega |F| \sin[k(L-x)]}{kT\cos(kL)} e^{i\omega t}$$

Again, the complex mechanical input impedance, Z^{input} of the driven string at the driving point, x = 0 is given by:

$$\begin{aligned} Z^{input} &\equiv \frac{F(t)}{u_y(x=0,t)} = \frac{kT\cos(kL)}{i\omega\sin(kL)} = -i\frac{kT}{\omega}\cot(kL) \\ &= -i\frac{T}{v_x}\cot(kL) = -i\sqrt{\mu T}\cot(kL) = -i\mu v_x \cot(kL) \\ &= -iZ_o \cot(kL) \end{aligned}$$

Note that here, for the driven free end-fixed end boundary conditions of this string, the complex mechanical input impedance, Z^{input} of the driven string at the driving point, x = 0 is *purely* *__reactive__* - i.e. it is purely imaginary!

The (complex) power, or time rate of energy transfer from the “free” end support at x = 0 to the string is given by:

$$\begin{aligned} P(t) &= \frac{\partial E(t)}{\partial t} = F(t)u_y^*(x=0,t) = -i\frac{\omega |F|^2 \sin(kL)}{kT\cos(kL)} \\ &= -i\frac{v_x |F|^2}{T}\tan(kL) = -i\frac{v_x |F|^2}{T\cot(kL)} = \frac{v_x |F|^2}{iT\cot(kL)} \\ &= \frac{|F|^2}{(Z^{input})^*} = \frac{|F|^2}{iZ_o\cot(kL)} \end{aligned}$$

Here, the power is (also) purely reactive - i.e. purely imaginary, or 90^o out of phase with the driving force. The physical meaning of this is the following. The driving force inputs energy into the string at x = 0, creating a right-moving traveling wave. The right-moving traveling wave propagates on the string until it reaches the fixed end at x = L, whereupon it is reflected and polarity-flipped, converted into left-moving traveling wave. The left-moving traveling wave then propagates back to x = 0, where it leaves the string, returning its associated energy to the power source. Thus, a given amount of energy is put into the vibrating string at x = 0 by the complex driving force, F(t), however a little while later, this same energy comes back and is returned to the driving force. Thus, for the equilibrium situation, no *net* energy is transferred to/from this string!

The driving force, F(t) has angular frequency, $\omega = 2\pi f$. In principle, the free end support at x = 0 can be driven at any frequency, f. Because of the free end-fixed end boundary conditions at x = 0 and x = L, respectively, at certain driving frequencies *resonances* (maximum amplitudes) and *anti-resonances* (minimum amplitudes) in the transverse vibrations of the string of length, L will occur. However, because of the free end-fixed end boundary conditions, the modes of