The most general form of the complex transverse displacement, y(x, t) of the standing wave is given by:

$$y(x,t) = y_{oR}e^{i(\omega t-kx)} + y_{oL}e^{i(\omega t+kx)}$$

where $y_{oR} = |y_{oR}|e^{i\delta}$ is the complex amplitude of the right-moving travelling wave and $y_{oL} = |y_{oL}|e^{i\delta}$ is the complex amplitude of the left-moving travelling wave.

At the driven end of the string, at x = 0, the complex driving force, F(t) must again balance against the transverse component of the tension, $T_y(x=0, t)$ at x = 0, which again for small amplitudes, is $T_y = T \sin\theta \cong T\,(\partial y(x=0, t)/\partial x)$, thus we must have

$$F(t) = |F|\,e^{i\omega t} = T\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=0} = T\left[-iky_{oR}e^{i\omega t} + iky_{oL}e^{i\omega t}\right] = -ikT\left[y_{oR} - y_{oL}\right]e^{i\omega t}$$

Thus, we see that:

$$|F| = -ikT\;\; y_{oL}\left[e^{+2ikL} + 1\right]$$

At the fixed end, x = L, the boundary condition on the transverse displacement is y(x=L,t) = 0. Thus,

$$y(x=L,t) = y_{oR}e^{i(\omega t-kL)} + y_{oL}e^{i(\omega t+kL)} = \left[y_{oR}e^{-ikL} + y_{oL}e^{ikL}\right]e^{i\omega t} = 0$$

This relation *must* hold for any/all times, t. Thus, we must have:

$$\left[y_{oR}e^{-ikL} + y_{oL}e^{+ikL}\right] = 0$$

i.e. this implies that $y_{oR}\, e^{-ikL} = -y_{oL}\, e^{+ikL}$, or that $y_{oR} = -y_{oL}\, e^{+2ikL}$. Plugging this result into the above driving force - tension balancing result, we see that

$$|F| = -ikT\left[y_{oR} - y_{oL}\right]$$

or:

$$y_{oR} = -y_{oL}e^{+2ikL} = -i\frac{|F|\,e^{+ikL}}{2kT\cos(kL)}$$

since $\cos(x) = ½\,[e^{+ix} + e^{-ix}]$, note also that $\sin(x) = ½i\,[e^{+ix} - e^{-ix}]$. Then we find that

$$y_{oL} = i\frac{|F|}{kT\left[1+e^{+2ikL}\right]} = i\frac{|F|\,e^{-ikL}}{kT\left[e^{-ikL}+e^{+ikL}\right]} = i\frac{|F|\,e^{-ikL}}{kT\left[e^{+ikL}+e^{-ikL}\right]} = i\frac{|F|\,e^{-ikL}}{2kT\cos(kL)}$$

Thus, the complex transverse displacement, *y*(*x*, *t*) of the driven string, at an arbitrary point, x, and time, *t* is given by:

$$\begin{aligned} y(x,t) &= y_{oR}e^{i(\omega t-kx)} + y_{oL}e^{i(\omega t+kx)} \\ &= -i\frac{|F|\,e^{i\omega t}}{2kT\cos(kL)}e^{ik(L-x)} + i\frac{|F|\,e^{i\omega t}}{2kT\cos(kL)}e^{-ik(L-x)} = \frac{|F|\sin[k(L-x)]}{kT\cos(kL)}e^{i\omega t} \end{aligned}$$