The (complex) transverse velocity, $u_y(x, t) = \partial y(x, t)/\partial t$ of the string at the position, x and at time, *t* is therefore:

$$u_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \frac{\omega\,|F|}{kT} e^{i(\omega t - kx)} = \frac{v_x\,|F|}{T} e^{i(\omega t - kx)}$$

Note that the complex transverse velocity, $u_y(x=0, t) = i\, v_x\, |F|/T$ is in phase with the driving force, $F(t) = |F|e^{i\omega t}$.

We now *define* the complex mechanical input impedance, $Z^{input}$ of the driven string (units of Newtons/(m/sec) = kg/sec) as the *ratio* of the complex driving force, F(t) to the complex transverse velocity, $u_y(x=0, t)$ at the driving point, *x* = 0:

$$Z^{input} \equiv \frac{F(t)}{u_y(x=0,t)} = \frac{T}{v_x}$$

In this situation, the complex mechanical input impedance of an infinitely long, driven string is a *purely real* quantity. This impedance is *purely "resistive"* - not "reactive" (i.e. it has no *imaginary* part), and is also known as the *characteristic* impedance, $Z_o = T/v_x = (\mu T)^{1/2} = \mu v_x$ of the infinitely long driven string.

The (complex) power, or time rate of energy transfer from the "free" end support at x = 0 to the string (units of Watts, or Joules/second) is defined by:

$$P(t) = \frac{dE(t)}{dt} \equiv F(t)u_y^*(x=0,t) = \frac{\omega\,|F|^2}{kT} = \frac{v_x\,|F|^2}{T} = \frac{|F|^2}{Z_o}$$

Where $u_y^*(x,y)$ is the complex conjugate of $u_y(x,t)$. In general, if $z = x + iy = |z|e^{i\varphi} = |z|[\cos\varphi + i\sin\varphi]$, then $z^* \equiv x - iy = |z|e^{-i\varphi} = |z|[\cos\varphi - i\sin\varphi]$. Here again, in this situation, the power is a *purely real* quantity. Note also that it has *no* time dependence. Thus, here in this situation the instantaneous power is *constant*. As a consequence of this the instantaneous power is *also* equal to the time-averaged power, here.

In general, the physical meaning of the *real* part of the complex power, P(t) is that Re(P(t)) is the rate of energy transferred from the power source to the string. We shall see below in the next example that the physical meaning of the *imaginary* part of the complex power, P(t) is that Im(P(t) is the rate of energy returned from the string back to the power source.

Next, we consider the (somewhat more physically realistic) case where the driven string has *finite* length (say, of length L). The string is again driven at x = 0 by the force $F(t) = |F|e^{i\omega t}$. We assume that the string is fixed at x = L (*i.e.* the end support at *x* = *L* is infinitely rigid = infinitely massive). For an acoustic guitar (and also for a hollow-body electric guitar), we imagine the bridge to be the driven, "free" end at x = 0, and the nut to be the fixed end at x = L, since the vibrations of the nut on an acoustic guitar are so much less than at the bridge. For a solid-body electric guitar, we imagine the nut to be the driven, "free" end at x = 0, and the bridge to be the fixed end at x = L, since the vibrations of the bridge on an electric solid-body guitar are so much less than at the nut. Then the right-moving travelling waves created by the driven, "free" end support at x = 0 are reflected and polarity-flipped at x = L and converted into left-moving travelling waves. Since the initial right-moving traveling wave is a continuous wave-train of sinusoidal, time-varying travelling waves, then when the reflected, left-moving travelling waves overlap with the right-moving traveling wave, a standing wave is generated on the driven string.