Non-Linear Effect of String Tension on a Non-Rigid End Support

Because of the fact that the string tension, T is not constant with time, if one (or both) of the end supports of the strings on a guitar - i.e. the bridge or the nut are not perfectly rigid, then since the strings pass over these not-completely rigid end supports at an angle, then the oscillating tension T(t) causes a transverse motion of these end supports, vibrating at a frequency, $2f_n$. This can excite the vibrational mode 2n, or in fact interfere (constructively or destructively, depending on the exact nature of the motion of the not-completely rigid end support), if the 2n vibrational mode is already present.

Since the angle at which the string passes over the end support varies as f_n, then there is also the possibility of exciting (and or interfering with) the vibrational mode 3n, with frequency $3f_n$.

Non-Linear Effect of String Tension on a Polarization of the String Vibration

If a vibrational mode, n of a string has initial polarization in the y-direction, vibrating in the (x,y) plane at frequency, f_n then this creates an oscillating/time-dependent tension, T(t) which has frequency, $2f_n$. The oscillating tension creates a time-dependent force on the vibrating string, but in the (x,z) plane, that is proportional to

$$|\, y_{on} \,|^2 \left(\cos\omega_n t + \cos 3\omega_n t\right)$$

This process is known as *parametric amplification*, because one of the parameters of the system (in this case the tension, T) provides the driving force. As the vibrational mode with z-polarization grows, it either takes energy from, or feeds back energy to the vibrational mode with y-polarization, depending on their relative phases.

The (initial) rate of rotation/precession of the polarization of the string Ω_n (units = radians/sec), for a single mode of vibration, n on a string, is given by:

$$\Omega_n = \alpha\left(\frac{Y_{string} A_{string}}{T}\right)\left(\frac{|\, y_{on} \,|^2 |\, z_{on} \,|^2}{L^2}\right)\omega_n$$

where α is a numerical factor of order of unity. Typically, the precession frequency of a guitar string, $f_n^{prec} = \Omega_n / 2\pi \sim 1$ Hz, though as can be seen from the above expression, it clearly depends on the string material, the string tension, T, the length of the string, L, the initial amplitude(s) in the y- and z- directions, and on the mode of vibration, n. Note that as the excitation of the string vibration decays with time, the rate of precession of the string also decreases with time.