

which has both a steady and an oscillating, time-dependent term. If the (overall) transverse displacement, $y(x,t)$ involves many modes of vibration, their contributions to the (overall) string tension are simply summed, because all cross-terms vanish during the integration in the above string-length formula. Then the overall tension increase, $\delta T(t)$ is given by:

$$\delta T(t) = \sum_{n=1}^{n=\infty} \delta T_n(t) = Y_{string} A_{string} \sum_{n=1}^{n=\infty} \left[\frac{n\pi^2 |y_{on}|^2}{8L^3} \right] (1 - \cos(2\omega_n t))$$

The predominant effect of this quasi-steady increase in string tension, $\delta T(t)$ is to raise the vibrational frequencies of *all* modes of vibration by the same factor - the fractional shift in frequency is given by:

$$\frac{\delta\omega(t)}{\omega} = \frac{\delta f(t)}{f} = \left(\frac{Y_{string} A_{string}}{L} \right) \left(\frac{\delta T(t)}{T} \right)$$

However, an additional shift in frequency for each mode arises because of the time-dependence of the tension. This causes a further (small) frequency shift for each mode of vibration,

$$\frac{\delta\omega_n(t)}{\omega_n} = \frac{\delta f_n(t)}{f_n} = 2 \left(\frac{Y_{string} A_{string}}{L} \right) \left(\frac{\delta T_n(t)}{T} \right)$$

which destroys the harmonicity of the modes of vibration - i.e. we no longer have the precise relation between the harmonics and the fundamental, $f_n = nf_1$, $n = 2, 3, 4, \dots$!

Thus, when a string of a guitar is plucked with a large initial transverse amplitude, as the string excitation decays with time, there is a slow glide in the pitch/frequency of vibration back toward its small-amplitude value. The degree of this pitch-glide, or “twang” depends on the square of the amplitude, the length, L tension, T and Young’s modulus of the string. Shorter-scale guitars with light gauge, low-tension strings, or guitars with de-tuned strings are more susceptible to such pitch-glide/twang effects.