Ultimately, all of the initial energy associated with the vibrating strings of the guitar winds up as heat, somewhere in the universe. Most of this initial energy is dissipated within the guitar itself. Only a small fraction of the initial energy of the vibrating string is converted into an electrical signal, which is then sent to the guitar amplifier. The voltage amplitude of the signal output from an electric guitar is typically on the order of $|V_{signal}|$ ~ 100 mV = 0.100 Volts (after initial, fast transients have died out). The (peak) power associated with this signal is $P_{signal} = |V_{signal}|^2/R_{load}$, where R_{load} is the magnitude of the input impedance of a guitar amplifier, typically 1 Meg-Ohm = 10^6 Ohms (note that this is also the nominal/industry standard input impedance of oscilloscopes). Thus, the peak power associated with a guitar signal is typically $P_{signal} \sim 10^{-8} = 10 \times 10^{-9}$ = 10 nano-Watts!

Non-Linear Effects in Vibrating Strings

Up to now, our discussions of various effects on string vibrations have all been associated with so-called *linear* processes. We now wish to discuss some non-linear effects of string vibrations. The first non-linear effect is a simple dependence of the natural frequency, f_n of the mode of vibration, n of the string upon its amplitude of vibration, $y_n(x,t)$.

Non-Linear Effect of String Tension

For a string of cross sectional area $A_{string} = \pi r2_{string}$, length, L, Young's modulus, Y_{string}, and material density, ρ_{string} stretched with tension, T between rigid/fixed end supports, located at x = 0 and x = L, the equation of motion for transverse waves on this string is given by:

$$\mu \frac{\partial^2 y(x,t)}{\partial t^2} = \rho_{string} A_{string} \frac{\partial^2 y(x,t)}{\partial t^2} = T \frac{\partial^2 y(x,t)}{\partial x^2}$$

where the x-direction is along the axis of the string and the y-direction is transverse to the axis of the string, e.g. in the horizontal plane - thus we say the string vibration is *polarized* in the (x,y) plane. The normal modes of vibration of transverse standing waves on the string are given by:

$$y_n(x,t) = | y_{on} | \sin\left(\frac{n\pi x}{L}\right) \cos \omega_n t$$

where $\omega_n = 2\pi f_n = k_n v_x = (n\pi/L)(T/\mu)^{1/2} = (n\pi/L)(T/\rho_{string}A_{string})^{1/2}$.

For a given mode of vibration of the string, n with transverse displacement, $y_n(x,t)$ the string is displaced from its equilibrium (i.e. zero-amplitude) configuration. The length of the string increases slightly, by an amount:

$$\delta L_n(t) = \int_{x=0}^{x=L} \left[1 + \left(\frac{\partial y_n(x,t)}{\partial x} \right)^2 \right]^{1/2} dx - L \cong \frac{n\pi^2 \, | y_{on} |^2}{8L^2} \left(1 - \cos(2\omega_n t)\right)$$

The string tension, T therefore increases by an amount, $\delta T_n(t) \cong Y_{string} A_{string} (\delta L_n(t)/L)$:

$$\delta T_n(t) = Y_{string} A_{string} \left[\frac{n\pi^2 \, | y_{on} |^2}{8L^3} \right] \left(1 - \cos(2\omega_n t)\right)$$