

The magnitude of the incremental force contribution,  $dF_{\text{Eddy}}(t)$  acting on an infinitesimal length,  $dx$  of the string due to the interaction of the induced Eddy current with the magnetic field,  $B(z=h, x)$  of the magnetic pole of the pickup at the location of the string is given by:  
The total force,  $F_{\text{Eddy}}(t)$  is obtained by summing up/integrating the incremental force

$$dF_{\text{Eddy}}(t) = I_{\text{Eddy}}(t)B(z = h, x) \sin \varphi \, dx = \frac{\varepsilon(t)}{R_{\text{string}}} B(z = h, x) \sin \varphi \, dx$$

contributions over the entire length of the vibrating string:

$$F_{\text{magnetic}}(t) = \int_{x=0}^{x=L} I_{\text{Eddy}}(t)B(z = h, x) \sin \varphi \, dx = \int_{x=0}^{x=L} \frac{\varepsilon(t)}{R_{\text{string}}} B(z = h, x) \sin \varphi \, dx$$

Again, because the magnetic field intensity decreases so rapidly in moving away in any direction from the immediate vicinity of the pole of the permanent magnet of the pickup, located directly underneath the guitar string, the bulk of the contribution of the integrand,  $(\varepsilon(t)/R_{\text{string}})B(x)\sin\varphi \, dx$  of the above integral occurs in the  $x$ -region of the string in the vicinity of the pole of the permanent magnet. Since  $\varepsilon(t) \cong u_y(x, t)B(x)\sin\varphi \, D$ , then  $F_{\text{magnetic}}(t) \cong u_y(x_{\text{pu}}, t)B^2(z=h, x_{\text{pu}})\sin\varphi \, D/R_{\text{string}}$ . Thus, since  $u_y(x, t) = \omega y_0 \sin(kx)\cos(\omega t)$ , this time-dependent magnetic force acting on the vibrating string depends linearly on the frequency,  $f$  of the vibrating string and the amplitude,  $y_0$  of the vibrating string, but depends quadratically on the magnetic field intensity at the location of the strings over the guitar pickup(s).

Even if the strings of an electric guitar are initially plucked horizontally, i.e. in the plane parallel to the strings of the guitar, the plane of the transverse displacement,  $y(x, t)$  of each of the strings will naturally begin to *precess* in both the horizontal and vertical direction with respect to the plane of the strings, with an angular precession frequency,  $\Omega_p$  that is much slower than the frequency,  $f$  of the fundamental mode of vibration of the string. This precession of the plane of polarization of the string arises due to a small non-linear effect associated with the transverse vibrations of the string affecting the tension,  $T$  in the string.

This magnetic force,  $F_{\text{Eddy}}(t)$  is maximum (zero) when the plane of the transverse displacement,  $y(x, t)$  is horizontal (vertical) - i.e. parallel (perpendicular) to the plane of the strings, where  $\varphi = 90^\circ$  ( $0^\circ$ ) and  $\sin\varphi = 1$  ( $0$ ), respectively.

There also exists *another* magnetic force due to the pickup(s) acting on the string(s) of an electric guitar. Since the strings of an electric guitar are magnetically permeable, the portion of the string(s) in proximity to the magnetic poles of the pickup(s) becomes magnetized, thus the string behaves as a linear magnetic quadrupole, as discussed above. Thus, when a (magnetized) electric guitar string undergoes transverse vibrations, with transverse amplitude,  $y(x, t)$  at a height,  $h$  above the permanent magnetic pole of a pickup, the separation distance,  $r$  between the (nearest) permanent magnetic pole of the electric guitar pickup (located directly under the equilibrium position of this string) and the string itself varies, as shown in the figure below: