However, for transverse bending waves in a string, the magnitude of the longitudinal velocity of propagation of such waves, v_x^{bend} is not a simple relation, as in the case for transverse waves on an "ideal" string (where we found $v_x = \omega/k$). Here, we find that

$$\left(v_x^{bend}\right)^2 = \omega K \sqrt{Y_{string} / \rho_{string}} = \omega K c_L \qquad with \qquad c_L \equiv \sqrt{Y_{string} / \rho_{string}}$$

Thus, the longitudinal wave speed for bending waves on a string, v_x^{bend} is proportional to the square root of the frequency! For wave propagation in which the wave speed is not independent of the frequency, but instead has a frequency dependence, we say that such wave propagation has *dispersion*.

A modified wave equation for a string with tension T and which has an additional term for describing the restoring force associated with bending stiffness of the string is given by:

$$\mu \frac{\partial^2 y(x,t)}{\partial t^2} = T \frac{\partial^2 y(x,t)}{\partial x^2} - YA_{string} K^2 \frac{\partial^4 y(x,t)}{\partial x^4}$$

This differential equation can be solved, e.g. for standing waves on a string of length L with fixed ends at $x = 0$ and $x = L$. The modes of vibration are as before:

$$y_n(x,t) = y_{on} \sin(n\pi x / L) \sin(\omega_n t)$$

where n = 1, 2, 3, 4, ... However, the frequencies of vibration of the various modes are given by:

$$f_n = n f_1^o \left[1 + \beta n^2\right]^{1/2}$$

where f_1^o is the fundamental frequency of the string without stiffness and the constant, β is given by:

$$\beta = \pi^2 YA_{string} K^2 \big/ TL^2$$

Finite stiffness in strings therefore raises the frequency of the string, slightly. Note that because of this, the harmonics of the fundamental will not be precisely integer multiples of the fundamental, for wavelengths that are integer fractions of the fundamental wavelength! Thus because the strings on a guitar have different thicknesses (gauges), there will be different amounts of stiffness associated with the strings. This is the primary reason for the need for separate adjustment of the bridge saddles on an electric guitar - in order for it to intonate properly when playing anywhere on the neck!

Effect of Motion of the End Supports on Vibrating Strings

On either an acoustic guitar or an electrical guitar, the strings contact the guitar at the nut (the zeroth fret) near the top of the neck of the guitar, and at the bridge of the guitar. On an acoustic guitar, the (wooden) bridge rests on the top plate (soundboard) of the guitar body, which is usually made of *e.g.* a thin sheet of spruce wood, supported by braces inside the guitar. The wooden bridge of an acoustic guitar may also have a thin strip of bone, ivory or synthetic