material and/or kinks in polymer chains (in the case of e.g. nylon strings, used in acoustic guitars). Typically, the ratio Y_2/Y_1 is often less than 10^{-4} in hard crystalline materials, such as metals - steel, etc, but can be as large as 10^{-1} =0.1 in some polymer materials. In general, this ratio is also temperature dependent.

Internal damping of string vibrations arising from these microscopic physical processes leads to an exponential decay time, $\tau_n^{internal}$ of the transverse displacement amplitude(s) for the different modes of vibrations of the string, $y_n(x,t) = y_o \exp(-t/\tau_n^{internal}) \sin(k_n x) \sin(\omega_n t)$:

This kind of damping is negligible in comparison to air damping for solid metal strings, but

$$\tau_n^{internal} = \frac{1}{\pi f_n}\frac{Y_1}{Y_2}$$

can be the dominant damping mechanism in gut or nylon strings used on acoustic and classical guitars. Note that this decay time is shortest at high frequencies.

There exist additional internal damping mechanisms. In metal strings, thermal conduction (i.e. conduction of heat) also results in damping of the string vibrations. However, again, this is a small effect. In compound strings - consisting of twisted fibers, or wound strings, there also exists internal friction due to the relative motion of the component parts. As strings age on the guitar, build-up of “grunge” on the strings from playing the guitar occurs - a combination of skin cells, finger grease, dirt and sweat, all of which can (and does) lead to significant damping of the string vibrations. These loss mechanisms are also frequency-dependent, in the same manner as the above-discussed loss mechanisms.

Energy Loss Through the End Supports of the Strings

As we have discussed above, if the bridge or nut on a guitar is not perfectly rigid, then at certain resonant frequencies, $f_n = v_x/\lambda_n = {}^{(2n-1)v_x}/_{4L}$, n = 1, 2, 3, 4, ... energy can be transferred from the mode, n of the vibrating string to the quasi-free end support. In a real guitar, this energy can then be transferred to places elsewhere in the guitar, with a commensurate loss in sustain of the vibrating guitar string at this frequency.

In considering the energy of a quasi-free end support, it is easier to work with the complex mechanical *admittance*, Y ≡ 1/Z rather than complex mechanical *impedance*, Z. The complex mechanical admittance, Y = G + iB, where the real part of the complex mechanical admittance, Re(Y) = G, is known as the mechanical *conductance*. The imaginary part of the complex mechanical admittance, Im(Y) = B is known as the mechanical *susceptance*.

If the complex mechanical impedance, Z = R + iX, where Re(Z) = R, known as the mechanical *resistance* and Im(Z) = X, known as the mechanical *reactance*, then it can be shown that:

$$G = \frac{R}{R^2 + X^2} = \frac{R}{|Z|^2} \qquad and \qquad B = \frac{-X}{R^2 + X^2} = \frac{-X}{|Z|^2}$$

and conversely, that:

$$R = \frac{G}{G^2 + B^2} = \frac{G}{|Y|^2} \qquad and \qquad X = \frac{-B}{G^2 + B^2} = \frac{-B}{|Y|^2}$$