The *net* viscous drag force, $F_{drag}(t)$ acting on the *entire* length, L of vibrating string at a given instant in time, *t* is obtained by summing up all the force contributions, $dF_{drag}(x,t)$ from each of the infinitesimal string segments, dx along the vibrating string, from $0 \le x \le L$:

$$F_{drag}(t) = \sum_{n=1}^{n=N} dF_{drag}(x,t) = \sum_{n=1}^{n=N} \frac{dF_{drag}(x,t)}{dx} dx$$

Going to the limit of a true infinitesimal, when $dx \to 0$, this sum converts to an integral representation:

$$F_{drag}(t) = \int dF_{drag}(x,t) = 2\pi f \rho_{air} A_{string} \left(\frac{\sqrt{2}}{M} + \frac{1}{2M^2} \right) \int_{x=0}^{x=L} u_y(x,t) dx$$

Similarly, the instantaneous power loss, $P_{drag}(t)$ for the entire length, L of vibrating string at time, t is given by:

$$P_{drag}(t) = \frac{dE_{drag}(t)}{dt} = \int dP_{drag}(x,t) = \int dF_{drag}(x,t) u_y(x,t) = 2\pi f \rho_{air} A_{string} \left(\frac{\sqrt{2}}{M} + \frac{1}{2M^2} \right) \int_{x=0}^{x=L} u_y^2(x,t) dx$$

For a given mode of vibration of the string, of frequency, f_n the transverse displacement amplitude will decay exponentially with time, *i.e.*

$$y_n(x,t) = y_{on}\; e^{-t/\tau_n} \sin(k_n x) \sin(\omega_n t)$$

The decay time constant, τ_n^{air} = the time for the transverse displacement amplitude, $y_{on}\, e^{-t/\tau n}$ associated with the harmonic, n of frequency, f_n to fall to $e^{-\tau n/\tau n} = e^{-1} = 1/e = 1/2.71828 = 0.36788$ of its initial value, y_{on} at $t = 0$.

The decay time constant associated with air damping associated with a given mode, n of vibration of the string is given by:

$$\tau_n^{air} = \frac{\rho_{string}}{2\pi \rho_{air} f_n} \left(\frac{2M_n^2}{2\sqrt{2} M_n + 1} \right)$$

where ρ_{string} is the *volume* mass density (kg/m^3) of the string, and

$$M_n = \tfrac{1}{2} r_{string}^2 \sqrt{2\pi} f_{on} / \eta^*_{air}$$

This decay time constant, τ_n^{air} is linearly proportional to ρ_{string}, but depends in a more complicated manner on the string radius and frequency. The decay time constant, $\tau_n^{air} \propto \rho_{string}\, r^2_{string}$ at low frequencies - independent of f_n, but $\tau_n^{air} \propto \rho_{string}\, r_{string} / \sqrt{f_n}$ at high frequencies. Thus, the decay time due to viscous air-damping of the strings will be shortest at high frequencies, affecting the higher harmonics first, and then successively lower harmonics from the time the strings are initially plucked.