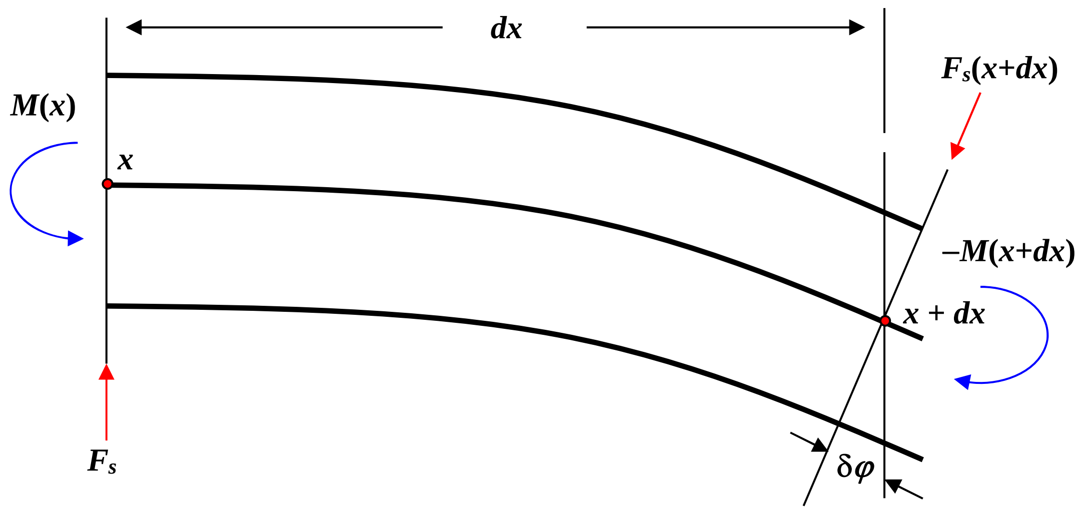

The shearing force, $F_s(x,t)$ is given by:

$$F_s(x,t) = \frac{\partial M(x,t)}{\partial x} = \frac{\partial}{\partial x}\left[-Y_{string} A_{string} K^2 \frac{\partial^2 y(x,t)}{\partial x^2}\right] = -Y_{string} A_{string} K^2 \frac{\partial^3 y(x,t)}{\partial x^3}$$

However, the shearing force, $F_s(x,t)$ acting on the string segment is not constant either. The net shearing force, $dF_s(x,t) = (\partial F_s(x,t)/\partial x)dx$ produces an acceleration perpendicular to the axis of the string. The equation of motion of the string segment, dx of mass, dm_{string} is given by Newton's second law, $dF_s(x,t) = dm_{string}\, a(x,t)$, were $a(x,t)$ is the acceleration of the string segment at the point x at time t:

$$\left(\frac{\partial F_s}{\partial x}\right) dx = dm_{string} \frac{\partial^2 y(x,t)}{\partial x^2} = \left(\rho_{string} A_{string}\, dx\right) \frac{\partial^2 y(x,t)}{\partial t^2}$$

$$-Y_{string} A_{string} K^2 \frac{\partial^4 y(x,t)}{\partial x^4} = \rho_{string} A_{string} \frac{\partial^2 y(x,t)}{\partial t^2}$$

$$-Y_{string} K^2 \frac{\partial^4 y(x,t)}{\partial x^4} = \rho_{string} \frac{\partial^2 y(x,t)}{\partial t^2}$$

This is a fourth-order differential equation, one which describes *bending* waves in a string (in the absence of tension in the string). The general solution of this differential equation has a complex transverse displacement, $y(x,t)$ of the form:

$$y(x,t) = \left[Ae^{kx} + Be^{-kx} + Ce^{ikx} + De^{-ikx}\right] e^{i\omega t}$$
$$= \left[A'\cosh(kx) + B'\sinh(kx) + C'\cos(kx) + D'\sin(kx)\right]\cos(\omega t + \varphi)$$

Thus, with four (apriori unknown) constants we must have four boundary conditions at the ends of the string in order to determine what these constants are. There are two boundary conditions at each end of the string. For fixed end supports, the boundary conditions at $x = 0$ and $x = L$ are $y(x,t) = 0$ and $a(x,t) = \partial^2 y(x,t)/\partial t^2 = 0$.