However, this does not mean that the vibrating string is not affected by the presence of the air around it. Indeed it is!

Nearly 150 years ago (in 1851), George Stokes [1] solved the problem of viscous air drag on a vibrating string. The viscous force on the string has two components. One is a mass-like load that *lowers* the vibrational mode frequencies of the string very slightly, the other force component produces an exponential decay of the amplitude with time. The frequency of vibration of the string in air is given by:

$$f_{air} = f_o\left[1-\left(\frac{\gamma}{f_o}\right)^2\right]^{1/2}$$

where f_o = frequency of vibration of the string in a vacuum; the parameter, γ is given by:

$$\gamma = f_o\rho_{air}\pi r_{string}^2\left(\frac{\sqrt{2}}{M}+\frac{1}{2M^2}\right) = f_o\rho_{air}A_{string}\left(\frac{\sqrt{2}}{M}+\frac{1}{2M^2}\right)$$

where ρ_{air} = density of air ≅ 1.205 kg/m^3, (at T = 20^o C, P = 1 Atm) $u_y(x,t)$ = transverse velocity of the string at the point x, at time, t; r_{string} = radius of string, $A_{string} = \pi r^2_{string}$, and:

$$M = \tfrac{1}{2} r_{string}^2\sqrt{2\pi}\, f_o / \eta^*_{air}$$

where $\eta^*_{air} \cong 1.52 \times 10^{-5}$ m^2/sec is the kinematic viscosity of air, i.e. $\eta^*_{air} = \eta_{air}/\rho_{air}$, where $\eta_{air} \cong 1.832 \times 10^{-5}$ kg/(m-sec) is the coefficient of viscosity of air. For a 0.010" diameter high-E string on a guitar, where $f_{hi\text{-}E}$ = 330 Hz, $M_{hi\text{-}E} \cong 0.44$. For a 0.046" diameter low-E string on a guitar, where $f_{lo\text{-}E}$ = 82 Hz, $M_{lo\text{-}E} \cong 2.35$. The value(s) of γ are extremely small, ranging from ~ 7.4×10^{-5} for the high-E string to ~ 4.8×10^{-5} for the low-E string. Thus, the frequency shifts due to viscous air drag effects on the vibrating strings of a guitar are exceedingly small, on the order of ~ 10^{-11} Hz.

For the range of string diameters and string vibration frequencies associated with stringed instruments, such as guitars, the viscous drag force, $dF_{drag}(x,t)$ acting on a infinitesimal segment of string of length dx, at the position, x along the string at time, t does so in such a way as to <u>always</u> <u>*oppose*</u> the motion of the string, i.e. <u>*retarding*</u> its motion, as the string passes through the air. This drag force, $dF_{drag}(x,t)$ acting at a point x on a vibrating string of length, L is given by:

The instantaneous power lost due to viscous damping of the vibrating string in air, $dP_{drag}(x,t)$

$$dF_{drag}(x,t) = 2\pi^2 f_o\rho_{air}u_y(x,t)r_{string}^2\left(\frac{\sqrt{2}}{M}+\frac{1}{2M^2}\right)dx = 2\pi f_o\rho_{air}\ u_y(x,t)A_{string}\left(\frac{\sqrt{2}}{M}+\frac{1}{2M^2}\right)dx$$

associated with the string segment, of length, dx at the point, x, at time t is given by:

The power loss, $dP_{drag}(x,t)$ is by definition the (time) rate of change of the total energy due to

$$dP_{drag}(x,t) = \frac{dE_{drag}(x,t)}{dt} = dF_{drag}(x,t) * u_y(x,t) = 2\pi f\rho_{air}u_y^2(x,t)A_{string}\left(\frac{\sqrt{2}}{M}+\frac{1}{2M^2}\right)$$

viscous air drag, $dE_{drag}(x,t)/dt$ associated with the string segment of infinitesimal length, dx, at the point x, at time, t. Note that since the viscous drag force, $dF_{drag}(x,t)$ acting on the string segment, dx is proportional to the transverse velocity of the string, $u_y(x,t)$ at the point x, at time, t, then the power loss and/or the rate of energy loss is proportional to the square of the transverse velocity, $u_y(x,t)$ at the point, *x* at time, *t*.