$$\tau_{damp} \cong \frac{1}{\omega}\frac{\left[1+1+\frac{1}{2}\left(\frac{b}{\mu\omega}\right)^2\right]^{1/2}}{\left[-1+1+\frac{1}{2}\left(\frac{b}{\mu\omega}\right)^2\right]^{1/2}} = \frac{1}{\omega}\frac{\left[2+\frac{1}{2}\left(\frac{b}{\mu\omega}\right)^2\right]^{1/2}}{\left[\frac{1}{2}\left(\frac{b}{\mu\omega}\right)^2\right]^{1/2}} = \frac{2}{\omega}\frac{\left[1+\left(\frac{b}{2\mu\omega}\right)^2\right]^{1/2}}{\left(\frac{b}{\mu\omega}\right)} = \frac{2\mu}{b}\left[1+\left(\frac{b}{2\mu\omega}\right)^2\right]^{1/2}$$

The frequency of vibration of transverse waves propagating in a viscous medium is also changed from its b = 0 value. If the amount of viscous damping is small, i.e. $b \ll \mu\omega$, then using the Taylor series expansion $(1+\varepsilon)^{1/2} \cong 1+\frac{1}{2}\varepsilon$ for $\varepsilon \ll 1$, it can be shown that:

$$\omega' \cong \omega\sqrt{1-\left(\frac{b}{2\mu\omega}\right)^2}$$

and:

$$f' \cong f\sqrt{1-\left(\frac{b}{4\pi\mu f}\right)^2}$$

## ***Air Damping Effects of Vibrating Strings***

In an electric guitar, or any stringed instrument there are in fact several dissipative mechanisms that are responsible for damping of the string vibrations. One such mechanism is due to viscous air-drag effects - the vibrating string(s) of a guitar do not occur in a vacuum - they occur in air, usually at atmospheric pressure (if at sea level). Microscopically, energy from the vibrating string is transferred to individual air molecules by their many collisions with the string as it vibrates. This is a statistical process. The vibrating string pushes air molecules out of the way as it vibrates - the air is essentially a viscous fluid, and has a viscous drag associated with it.

A string with either a transverse travelling wave or a transverse standing wave vibrates (transversely) back and forth in a plane. As it does so, it produces a compression wave in the air in front of it, and a rarefaction of the air behind it. From this, one might think that this would enable the vibrating string to couple extremely well to the air, and therefore act as a good radiator of sound waves. However, this is not the case, because the diameter of the string is so small in comparison to the wavelength(s) of sound in air.
For example, electric guitar strings have diameters in the range *e.g.* ~ 0.009"–0.046" (*i.e.* 0.2286–1.1684 mm). The speed of sound in air, at sea level is $v_s$ ~ 330 m/s. The high-E string of a guitar has frequency $f_{hi\text{-}E} = 330$ Hz. Thus, the wavelength in air associated with playing an open high-E string on a guitar is $\lambda_{hi\text{-}E} = v_s/f_{hi\text{-}E} = 1\ m$, which is much much greater than the diameter of the high-E string on a guitar. Since sound waves do indeed consist of compression & rarefaction of the air (i.e. local density perturbations in the air), the compression & rarefaction of air on the distance scale of the diameter of strings of a guitar is very inefficient at transferring sound energy from the string to the surrounding air - essentially at this distance scale, the air in the compressed region immediately ahead of the string, being pushed (*i.e.* compressed) by the vibrating string simply flows around the string to the backside of the string, where the air pressure is lower, thus cancelling out the sound even before it has time to radiate away!