and:

$$k'_I = \omega\left(\frac{\mu}{2T}\right)^{1/2}\left[-1+\sqrt{1+\left(\frac{b}{\mu\omega}\right)^2}\,\right]^{1/2}$$

Now since:

$$y(x,t) = y_o e^{i(\omega t - k'x)} = y_o e^{i\omega t} e^{-ik'x} = y_o e^{i\omega t} e^{-i(k'_R - ik'_I)x} = y_o e^{i\omega t} e^{-ik'_R} e^{-k'_I x} = y_o e^{i(\omega t - k'_R x)} e^{-k'_I x}$$

we see that the amplitude of the transverse displacement, y(x,t) associated with a right-moving transverse traveling wave on the string, vibrating in a viscous, dissipative medium will be attenuated to 1/e of its initial value (here assuming the wave originates at x = 0) in a characteristic distance $x = 1/k'_I$. We therefore define this characteristic damping distance as the so-called attenuation length, $\lambda_{atten} \equiv 1/k'_I$:

$$\lambda_{atten} \equiv \frac{1}{k'_I} = \frac{\frac{1}{\omega}\left(\frac{2T}{\mu}\right)^{1/2}}{\left[-1+\sqrt{1+\left(\frac{b}{\mu\omega}\right)^2}\,\right]}$$

The *phase speed*, v'_x (= longitudinal propagation speed) of this viscously damped right-moving traveling wave is no longer $v'_x = T/\mu$, because of the presence of damping. It is now $v'_x = \omega/k'_R$. Note that when b = 0 (i.e. no viscous damping of the string) then $k = k'_R = \omega/v_x$ (since then $v'_x = v_x = (T/\mu)^{½}$), and $k'_I = 0$. Thus,

$$v'_x = \frac{\omega}{k'_R} = \frac{\left(\frac{2T}{\mu}\right)^{1/2}}{\left[1+\sqrt{1+\left(\frac{b}{\mu\omega}\right)^2}\,\right]^{1/2}}$$

Thus, we see that in the presence of viscous damping ($b \neq 0$) the phase speed/longitudinal wave speed, $v'_x < v_x = (T/\mu)^{½}$.

The characterstic damping time, $t = \tau_{damp}$ in which the amplitude of the transverse wave is exponentially damped to $1/e = 1/e^{+1} = e^{-1} = 1/2.71828 = 0.36788$ of its initial value is given by:

$$\tau_{damp} = \frac{\lambda_{atten}}{v'_x} = \frac{1}{k'_I} \Big/ \frac{\omega}{k'_R} = \frac{1}{\omega}\left(\frac{k'_R}{k'_I}\right) = \frac{1}{\omega}\frac{\left[1+\sqrt{1+\left(\frac{b}{\mu\omega}\right)^2}\,\right]^{1/2}}{\left[-1+\sqrt{1+\left(\frac{b}{\mu\omega}\right)^2}\,\right]^{1/2}}$$

Note that this damping time is valid not only for transverse traveling waves, but also for transverse standing waves. If the amount of viscous damping is small, i.e. $b \ll \mu\omega$, then using the Taylor series expansion $(1+\varepsilon)^{½} \cong 1 + ½\varepsilon$ for $\varepsilon \ll 1$, the damping time, τ_{damp} is: