or:

$$\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = +\frac{b}{T}\frac{\partial y(x,t)}{\partial t}$$

or:

$$\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{1}{v_x^2}\frac{\partial^2 y(x,t)}{\partial t^2} = +\frac{b}{T}\frac{\partial y(x,t)}{\partial t}$$

Mathematically, this wave equation, which describes the (viscously) damped vibrations of transverse waves on a stretched string is a so-called second-order, linear, but (now) *inhomogeneous* differential equation. This 2nd order linear differential equation is inhomogeneous because it has a term which is a single derivative of y(x,t) - the velocity-dependent damping term on the right-hand side of the above equation.

The solution, y(x,t) of this 2nd order linear inhomogeneous wave equation that describes the behavior of e.g. viscously-damped right-moving traveling waves on the string is of the form: Plugging this solution in to the above wave equation, we obtain the so-called *characteristic*

$$y(x,t) = y_o e^{i(\omega t - k' x)}$$

equation:

$$k'^2 = \omega^2\left(\frac{\mu}{T}\right)\left[1 - i\frac{b}{\mu\omega}\right]$$

Thus, it can be seen that the wavenumber, k' associated with the viscously damped, right-moving traveling wave is complex - i.e. it has a real part, k'_R and an imaginary part, k'_I:

$$k' = k'_R - ik'_I$$

Note that the minus sign in the above formula is extremely important - for a right-moving traveling wave, the amplitude of this wave is exponentially damped as x increases (for a left-moving traveling wave, the amplitude is exponentially damped as x decreases).

Then $k'^2 = k'^2_R - 2ik'_R k'_I - k'^2_I$, we plug this into the above characteristic equation, identify the resulting terms that are purely real, and those that are purely imaginary:

$$k'^2_R - k'^2_I = \omega^2\left(\frac{\mu}{T}\right) \qquad \textit{and} \qquad -2ik'_R\, k'_I = i\omega^2\left(\frac{\mu}{T}\right)\left[\frac{b}{\mu\omega}\right] = i\omega\frac{b}{T}$$

We can then solve the two resulting quadratic equations for k'_R and k'_I with the physical constraint(s) that both k'_R and k'_I must be positive, and after some algebra, we obtain:

$$k'_R = \omega\left(\frac{\mu}{2T}\right)^{1/2}\left[1 + \sqrt{1 + \left(\frac{b}{\mu\omega}\right)^2}\right]^{1/2}$$