independent of the physical origin of the damping process, they will have an impact on the frequenc(ies) associated with the string vibrations. There are various physical mechanisms associated with dissipative losses of the string's initial energy/damping of the string's vibrations which we will discuss in turn, below. However, first we wish to discuss damping of the string vibrations in a general manner.

Consider a stretched string of length *L* and mass per unit length, μ completely immersed in a viscous medium that provides a damping force, *F* on the string that is propotional to the transverse velocity of the string, $u_y(x,t)$.

The (generic) damping force, $dF_{damp}(x,t)$ acting on an infinitesimal element, or segment, dx of the string at the point, x along the string at time, t is given by:

$$dF_{damp}(x,t) = -b\ dx\ u_y(x,t) = -b\ dx \frac{\partial y(x,t)}{\partial t}$$

where b is the constant of proportionality for the damping force. The minus sign indicates that the damping force always acts in such a way as to oppose the motion of the vibrating string.

The wave equation describing an infinitesimal element, *dx* of vibrating string in the *absence* of viscous damping, for small amplitude vibrations of the string is given by:

$$\mu\ dx \frac{\partial^2 y(x,t)}{\partial t^2} = T\{\sin(\theta + d\theta) - \sin\theta\} = T\ \ d\sin\theta = T\cos\theta\ \ d\theta \cong T\frac{\partial\theta}{\partial x}dx \cong T\frac{\partial^2 y(x,t)}{\partial x^2}dx$$

or:

$$\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = 0$$

or:

$$\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{1}{v_x^2}\frac{\partial^2 y(x,t)}{\partial t^2} = 0$$

Mathematically, this wave equation, which describes the undamped vibrations of transverse waves on a stretched string is a so-called second-order, linear, *homogeneous* differential equation. It is a *second-order* differential equation because it has *double derivatives* of y(x,t), one of position, x ($\partial/\partial x^2$) and the other, of time, t ($\partial/\partial t^2$). It is a *linear* differential equation, because all terms in this equation have a *linear* dependence on y(x,t). It is a *homogeneous* differential equation because there are *only* the double-derivatives in this 2^{nd} order linear differential equation - not single, or triple, quadruple etc. derivatives.

In the presence of viscous damping of the string, the wave equation is modified:

$$\mu\ dx \frac{\partial^2 y(x,t)}{\partial t^2} = T\frac{\partial^2 y(x,t)}{\partial x^2}dx - bdx\ u_y(x,t) = T\frac{\partial^2 y(x,t)}{\partial x^2}dx - bdx\frac{\partial y(x,t)}{\partial t}$$