Suppose the string with tension, *T* has a rigid, fixed end support at $x = 0$, but at $x = L$, this end support is not completely rigidly fixed, but instead, this end support behaves as if it is quasi-free, with a finite mass, *M*. Note that a perfectly rigid, fixed end support can be thought of as having infinite mass, $M = \infty$. Then as the string vibrates transversely, for small-amplitude vibrations, it will exert a transverse force, $F_y(t) \cong -T\,(\partial y(x=L,t)/\partial x)$ on this quasi-free end support at x = L, of mass, M. By Newton's second law (F = Ma), we have:
Note that the acceleration, a of the mass, M at the end support located at x = L is the same as the

$$F_y(t) = -T\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=L} = Ma = M\left.\frac{\partial^2 y(x,t)}{\partial t^2}\right|_{x=L}$$

transverse acceleration of the string, $a_y(x=L) = \partial^2 y(x=L,t)/\partial t^2$ at this end support. This force will in general be complex.

The (complex) transverse displacement of the string at an arbitrary point, x along its length, L at a given time, t is again given by:

$$y(x,t) = y_{oR}e^{i(\omega t - kx)} + y_{oL}e^{i(\omega t + kx)}$$

with $y_{oR} = |y_{oR}|e^{i\delta}$ and $y_{oL} = |y_{oL}|e^{i\delta}$.We apply the fixed-end boundary condition at x = 0, namely that y(x=0,t) = 0, where:

$$y(x=0,t) = y_{oR}\left.e^{i(\omega t - kx)}\right|_{x=0} + y_{oL}\left.e^{i(\omega t + kx)}\right|_{x=0} = y_{oR}e^{i\omega t} + y_{oL}e^{i\omega t} = \left(y_{oR} + y_{oL}\right)e^{i\omega t} = 0$$

This can only be satisfied for any/all time(s) t, if $y_{oR} = -y_{oL}$. Then the transverse displacement, y(x,t) of the string, for an arbitrary point, *x* and time, *t* is:

$$y(x,t) = y_{oR}\left(e^{-ikx} - e^{+ikx}\right)e^{i\omega t} = -y_{oR}\left(e^{+ikx} - e^{-ikx}\right)e^{i\omega t} = -2i\;y_{oR}\sin(kx)e^{i\omega t}$$

Then:

$$u_y(x,t) = \frac{\partial y(x,t)}{\partial t} = 2\omega y_{oR}\sin(kx)e^{i\omega t}$$

and:

$$\frac{\partial y(x,t)}{\partial x} = -2iky_{oR}\cos(kx)e^{i\omega t}$$

and:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = +2i\omega^2 y_{oR}\sin(kx)e^{i\omega t}$$

Inserting these relations into the above force relation at $x = L$, we have:

$$F(t) = +2iky_{oR}T\cos(kL)e^{i\omega t} = 2iM\omega^2 y_{oR}\sin(kL)e^{i\omega t}$$