The (complex) power, or time rate of energy transfer from the "free" end support at $x = 0$ to the string (or vice versa, for the time-reversed situation), at the resonant frequency, $f_n = v_x/\lambda_n = {}^{(2n-1)V_x}/_{4L}$, n = 1, 2, 3, 4, ... is given by:

$$P_n(t) = \frac{dE_n(t)}{dt} \equiv F(t)u_{ny}^*(x=0,t) = -i\frac{\omega_n \,|\,F\,|^2 \,\sin(k_n L)}{k_n T\cos(k_n L)}$$
$$= -i\frac{v_x\,|\,F\,|^2}{T}\tan(k_n L) = -i\frac{v_x\,|\,F\,|^2}{T\cot(k_n L)} = \frac{v_x\,|\,F\,|^2}{iT\cot(k_n L)}$$
$$= \frac{|\,F\,|^2}{(Z_n^{input})^*} = \frac{|\,F\,|^2}{iZ_o\cot(k_n L)} = \frac{|\,F\,|^2}{iZ_o\cot\left[\frac{(2n-1)\pi}{2}\right]} = \infty$$

Of course, since we are driving the string with a finite force, we can't possibly expect to have an infinite amount of energy/power returned to the power source, if it wasn't put in, in the first place. The reason for this result is that we have implicitly assumed in this situation that the *width* of the resonances, f_n are infinitely narrow, which is in fact not the case for real strings on a guitar - the widths of the resonances are finite.

If the complex mechanical input impedance, $Z_n^{input} = 0$ at a succession of resonant driving frequencies, $f_n = v_x/\lambda_n = {}^{(2n-1)V_x}/_{4L}$, n = 1, 2, 3, 4, ... how does the complex mechanical input impedance, Z^{input} behave *in-between* such resonances, say in- between the resonant frequencies $f_n = {}^{(2n-1)V_x}/_{4L}$ and $f_{n+1} = = {}^{(2n+1)V_x}/_{4L}$?

When the frequency, f of the complex driving force, F(t) is precisely at a frequency, $f_m = mv_x/2L$ for m = 1, 2, 3, 4, ... the complex mechanical input impedance, Z_m^{input} of the driven string at the driving point, x = 0 will be $Z_m^{input} = \pm\, i\, \infty$, because then $\cot(m\pi) = \pm\,\infty$ for m = 1, 2, 3, 4, ... The *magnitude* of the complex mechanical input impedance of the driven string, $|Z_m^{input}| = \infty$ when $f = f_m = mv_x/2L$ for m = 1, 2, 3, 4, ... These frequencies correspond to so-called *anti-resonances*. The free end support is vibrating up and down, transverse to the string at the frequency $f = f_m = mv_x/2L$ for m = 1, 2, 3, 4, ... but because the complex mechanical input impedance has magnitude, $|Z_m^{input}| = \infty$ at such frequencies, no energy can be transferred by the driving force, F(t) from the driven free end of the string to the string at these anti-resonant frequencies!

Again, by time reversal invariance, if a plucked string has a fundamental, or one of the higher harmonics vibrates at one of these *anti-resonant* frequencies, $f_m = mv_x/2L$ for m = 1, 2, 3, 4, ... of the "free" end support, there will be *no* energy transferred from the vibrating string to the "free" end support at such a frequency, thus the *sustain* of the string will be as long as in the case of fixed, perfectly rigid end supports at both ends of the string!

The complex power, P(t) for the anti-resonant frequencies, $f_m = mv_x/2L$ for m = 1, 2, 3, 4, ... is given by:

$$P(t) = \frac{\partial E(t)}{\partial t} = F(t)u_y^*(x=0,t) = \frac{|\,F\,|^2}{(Z^{input})^*} = \frac{|\,F\,|^2}{iZ_o\cot(k_m L)} = \frac{|\,F\,|^2}{iZ_o\cot(m\pi)} = 0$$

The following figure shows a plot of the quantity $-i\, Z^{input} = (T/v_x)\cot(kL)$ vs. kL for the case of free end-fixed end supports at x = 0 and x = L, respectively, with $(T/v_x) = 1.0$.