where Y_{string} = Young's modulus (ratio of stress/strain) associated with the material of the string (stress is the force per unit area that creates the deformation of the string) and $dA_{filament}$ is the cross sectional area of the filament located a distance z from the neutral axis.

The moment of this force about the neutral axis is given by

$$dM = dF \ z = \left[Y_{string} \, z \frac{\partial \varphi}{\partial x} dA_{filament} \right] z$$

Thus, the total moment to compress/stretch all the filaments in the string is given by:

$$M = \int dM = Y_{string} \frac{\partial \varphi}{\partial x} \int z^2 dA_{filament}$$

The so-called radius of gyration, K of the cross section of the string is defined by:

$$K^2 \equiv \frac{1}{A_{string}} \int z^2 dA_{filament}$$

where $A_{string} = \pi r^2_{string}$ is the cross sectional area of the string. For a cylindrical string of radius r_{string}, the radius of gyration of the string is $K = r_{string}/2$.

The bending moment, $M(x,t)$ of the string segment dx at the point x and time t is thus:

$$M(x,t) = Y_{string} \frac{\partial \varphi(x,t)}{\partial x} A_{string} K^2 \cong -Y_{string} A_{string} K^2 \frac{\partial^2 y(x,t)}{\partial x^2}$$

since $\partial\varphi(x,t)/\partial x \cong -(\partial^2 y(x,t)/\partial x^2)$ for small angles $\delta\varphi$ (i.e. small amplitude vibrations of the string).

The bending moment, $M(x,t)$ is not the same for every portion of the string segment. In order to keep the string in equilibrium a shearing force, F_s must exist with bending moment $dM(x,t) = F_s(x,t)dx$ as shown for a "snapshot" of the string segment, dx in the figure below: