***Waves II*:**

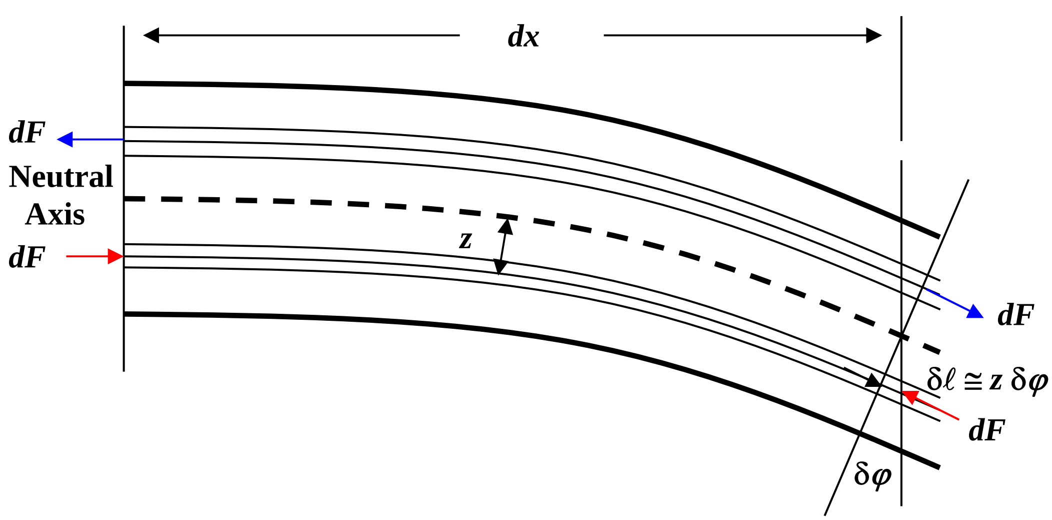
Vibrations of Real Strings

Thus far, our discussion(s) of wave propagation on strings has treated them in an idealized manner - *i.e*. that strings, while able to withstand a tension *T*, are perfectly elastic/flexible entities, and have no dissipative losses and/or non-linear properties associated with them. We made these simplifications in order to gain an understanding of the basic phenomena associated with wave propagation on strings. Now we wish to refine our treatment of string vibrations, in order to achieve a more physically realistic mathematical description of string vibrations.

Vibrations of Stiff Strings

Real vibrating strings on a guitar have a restoring force which is primarily due to the string tension, *T*. However, on heavier-gauge strings, the restoring force is also due to the stiffness of the string - it is not perfectly elastic. The wave equation we obtained in the previous lecture notes for vibrating strings did not take into account the possibility that the string could have a stiffness associated with it.

When a string of radius r_{string} bends in the process of vibrating (for small amplitude vibrations), the outer portion of the string is stretched slightly while the inner portion of the string is compressed slightly - the material of the string is in fact an elastic solid. Somewhere in between the outer and inner parts of the string exists a so-called neutral axis, whose length remains unchanged. A filament of the string located a distance z above (below) the neutral axis is stretched (compressed) by an amount $\delta\ell \cong z\ \delta\varphi$ as shown in the figure below, for a small segment (of infinitesimal length dx) of the string:



The strain, *S* is the fractional change in length of the string segment, *i.e*. the strain is the ratio $S = \delta\ell/dx \cong z\ \delta\varphi/dx$. The incremental amount of force, *dF* required to produce this strain is:

$$dF = Y_{string} S dA_{filament} \underset{1}{=} Yz \frac{\partial \varphi}{\partial x} dA_{filament}$$