The Principle of Linear Superposition of Waves

The principle of linear superposition is a very powerful one. Mathematically, if $y_1(x,t)$ and $y_2(x,t)$ are both solutions to the wave equation, then it can be shown that the linear combination, $y_{tot}(x,t) = y_1(x,t) + y_2(x,t)$ is *also* a solution to the wave equation. Thus, if individual kinds of transverse traveling waves, $y_1(x,t)$ and $y_2(x,t)$ can propagate on a string, then it is also possible for the wave $y_{tot}(x,t) = y_1(x,t) + y_2(x,t)$ to propagate on this same string. We will see throughout this course, that the principle of linear superposition of waves manifests itself in many ways, and has many different consequences. In general, we could have:

$$y_{tot}(x,t) = y_1(x,t) + y_2(x,t) + y_3(x,t) + y_4(x,t) + ... = \sum_{n=1}^{n=\infty} y_n(x,t)$$

However, here we must remind the reader that one of our original constraints in deriving the wave equation was that the amplitudes of the wave(s) on the string were *small*. As long as the total/overall amplitude, $y_{tot}(x,t)$ resulting from superposing arbitrarily many individual waves is small, then the above mathematics is a good description of the physical system. If this is *not* true, then one needs to develop a more sophisticated theoretical description, valid for large-amplitude string vibrations. In this situation, use of the above formulae will be inaccurate.

Reflection of Waves at a Boundary

What happens to a transverse traveling wave when it reaches a boundary, such as the end of a taught string? The answer depends on whether the end of the string is rigidly fixed (i.e. immovable) or is able to freely undergo transverse vibrations, under the transitory influences (i.e. forces) associated with the transverse traveling wave when it reaches this end point.

A.) Reflection of Waves at a Fixed End

If the end of a taught string, *e.g*. at $x = L$ is rigidly/immovably fixed, then mathematically, this means that the transverse displacement, $y(x = L, t) = 0$ at the point $x = L$ for *any* time t. This mathematical statement of the (fixed) end condition, is generically known as a *boundary condition* associated with the transverse waves propagating on the string. An electric solid-body guitar or bass with a rigidly-mounted bridge attached to the body of the guitar is an example of fixed-end boundary condition for the reflection of transverse traveling waves on the strings of the guitar - or at least a good, first-order approximation of what actually happens.

If we have a right-moving transverse traveling wave in the shape e.g. of a gaussian pulse, described mathematically as $y(x,t) = y_o \exp\{-(x-v_x t)^2\}$, then as this waveform reaches the fixed endpoint at $x = L$, the boundary condition $y(x=L, t) = 0$ must be obeyed. As this pulse impinges on the fixed end support, the only way that $y(x=L,t) = 0$ can be obeyed is if this pulse is simultaneously reflected and polarity inverted. Reflection means that the right-moving gaussian-shaped transverse traveling wave, $y(x,t) = y_o \exp\{-(x-v_x t)^2\}$ is converted into a left-moving gaussian-shaped transverse traveling wave, $y(x,t) = y_o \exp\{-[(x-2L)+v_x t]^2\}$ at the point $x = L$; polarity inversion means that the left-moving