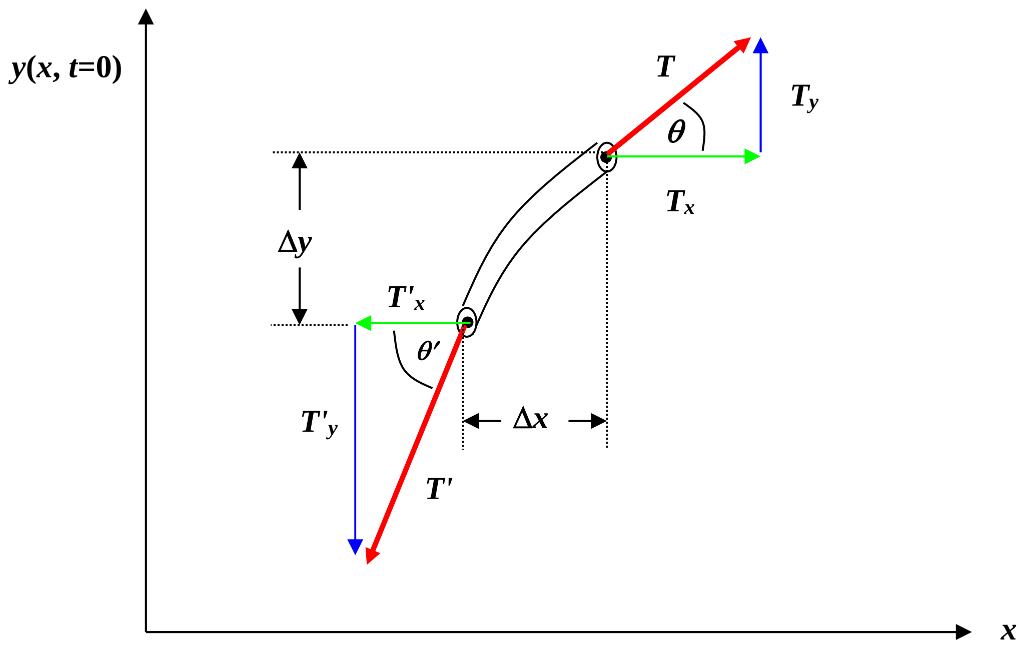
The force(s) acting on an infinitesimally small segment of the string, at $t = 0$ are shown in the figure below:



In the above figure, we have resolved the (vector) tensions, T and T' acting on each end of the infinitesimal segment of the string, into their x- and y-components. On the upper right-hand end of the string segment, $T_x = +|T| \cos\theta$, and $T_y = +|T| \sin\theta$. Note that the *magnitude* of the tension T, $|T| = (T_x^2 + T_y^2)^{1/2}$, since $\cos^2\theta + \sin^2\theta = 1$. On the lower left-hand end of the string segment, $T'_x = -|T'| \cos\theta'$, and $T'_y = -|T'| \sin\theta'$, with the *magnitude* of the tension T', $|T'| = (T'^2_x + T'^2_y)^{1/2}$, again since $\cos^2\theta' + \sin^2\theta' = 1$.

In the horizontal (x-direction), for small-amplitude transverse waves, there is no *net* force acting on the string - the horizontal tension components T_x and T'_x balance (*i.e.* cancel) each other. Newton's second law, $F_x = ma_x$ says that if there were a net force, F_x in the x-direction, then this string segment, of mass, m would accelerate in that direction, with acceleration, a_x. Since the string segment does not accelerate *longitudinally* for *transverse* waves, there is no net *longitudinal* force acting on the string segment. Mathematically, this is stated as $F_x = T_x + T'_x = 0$, *i.e.* $T_x = -T'_x$, or equivalently, $|T| \cos\theta = +|T'| \cos\theta'$.

In the vertical (y-direction), for small-amplitude transverse waves, there *is* a net restoring force, F_y acting on the string, which acts in such a way as to move the string back towards its equilibrium ($y = 0$) position, for a non-zero transverse displacement of the string, $y(x,t)$. This net transverse restoring force must be present, otherwise the string wouldn't vibrate - transversely! The vertical tension components, T_y and T'_y therefore do not cancel each other completely. Here, Newton's second law, $F_y = ma_y = T_y + T'_y \neq 0$ says that there is a net acceleration, a_y in the vertical (y-direction), transverse to the axis of the string. Then $F_y = ma_y = T_y + T'_y = |T| \sin\theta - |T'| \sin\theta' \neq 0$.