3. For a right-moving gaussian traveling wave, $y(x, t) = y_o \exp\{-(x - v_x t)^2\}$, with amplitude, $y_o = 1\ m$ (= transverse displacement of the string from its equilibrium position), and *longitudinal* wave velocity, $v_x = +1\ m/sec$, show that the partial derivative of $y(x, t)$ with respect to x, $\partial y(x, t)/\partial x$ (which physically is the local slope of the string at the point x, at a given time t), is given by the expression $\partial y(x, t)/\partial x = -2(x - v_x t)\, y_o \exp\{-(x - v_x t)^2\} = -2(x - v_x t)\, y(x, t)$. Then, using *e.g.* a spread-sheet program, such as Excel, or other mathematical software, make plot(s) of the transverse velocity of the right-moving gaussian traveling wave on the string, $u_y(x, t) = -(\partial y(x, t)/\partial x)\, v_x$, as a function of position, x, for times $t = -5$, 0 and +5 seconds.

4. Show that a traveling wave (either right- or left-moving), $y(x, t) = f(x \mp v_x t)$ satisfies the wave equation, $\partial^2 y(x,t)/\partial x^2 = 1/|v_x|^2\ \partial^2 y(x,t)/\partial t^2$ by explicitly carrying out the differentiation on both sides of the equation, using the chain-rule of differentiation.

5. After you have read through, learned and understood the contents of the lecture notes on Fourier analysis of waveforms, then come back to these lecture notes and derive the relation(s):

$$y_n(x,t) = C_n \cos(\omega_n t + \delta_n)\sin(k_n x) = C_n \sin(\omega_n t + \delta_n')$$

$$y_n(x,t) = C_n \cos(\omega_n t')\sin(k_n x)$$

$$y_n(x,t) = C_n \sin(\omega_n t^*)\sin(k_n x)$$

from the relation:

$$y_n(x,t) = [A_n \cos(\omega_n t) + B_n \sin(\omega_n t)]\sin(k_n x)$$

References for Waves and Further Reading:

1. The Physics of Musical Instruments, 2nd Edition, Neville H. Fletcher and Thomas D. Rossing, Springer, 1997.

2. The Acoustical Foundations of Music, John Backus, W.W. Norton and Company, 1969.

3. Mathematical Methods of Physics, 2nd Edition, Jon Matthews and R.L. Walker, W.A. Benjamin, Inc., 1964.