The time-averaged power, $<P(t)>$ associated with the nth normal mode of vibration of the entire string, of length, L in a standing wave is given by

$$\left\langle P_n(t)\right\rangle \equiv \frac{1}{\tau_n}\left[\left\langle KE_n(t)\right\rangle + \left\langle PE_n(t)\right\rangle\right] = \frac{1}{\tau_n}\left\langle E_n(t)\right\rangle = \frac{1}{\tau_n}\left[\tfrac{1}{8} m\omega_n^2 C_n^2\right] = \tfrac{1}{2} P_n$$

Here again, the time-averaged power, $<P_n>$) associated with the n^{th} normal mode of vibration of the entire string, of length, L is *not* the same as the instantaneous power, P_n because of how it is defined - the time averages of the kinetic and potential energy for each mode, $<KE_n(t)>$ and $<PE_n(t)>$ are carried out *separately*. Thus, P_n is also known as the *peak* power, while $<P_n> = ½\, P_n$ is known as the *root-mean-square*, or *rms* power.

If there are many normal modes of oscillation present as standing waves on a string, then the total power, P_{tot} summing over all modes is given by:

$$P_{tot} = \sum_{n=1}^{n=\infty} P_n = \sum_{n=1}^{n=\infty} P_n(t) = \sum_{n=1}^{n=\infty}\left(\frac{1}{\tau_n}\right)\left[KE_n(t) + PE_n(t)\right] = \sum_{n=1}^{n=\infty}\left(\frac{1}{\tau_n}\right)\left[\tfrac{1}{4} m\omega_n^2 C_n^2\right] = \sum_{n=1}^{n=\infty}\left(\frac{1}{\tau_n}\right)E_n$$

The time-averaged total power, $<P_{tot}>$ summing over all modes is given by:

$$< P_{tot} > = \sum_{n=1}^{n=\infty} < P_n(t) > \equiv \sum_{n=1}^{n=\infty}\left(\frac{1}{\tau_n}\right)\left(< KE_n(t) > + < PE_n(t) >\right) = \sum_{n=1}^{n=\infty}\left(\frac{1}{\tau_n}\right)\left[\tfrac{1}{8} m\omega_n^2 C_n^2\right] = \sum_{n=1}^{n=\infty}\left(\frac{1}{\tau_n}\right) < E_n >$$

Note that the time-averaged total power, $<P_{tot}>$ is *not* the same as the instantaneous power, P_{tot} because of how it is defined - the time averages of the kinetic and potential energy for each mode, $<KE_n(t)>$ and $<PE_n(t)>$ are carried out *separately*. Thus, P_{tot} is also known as the *peak* total power, while $<P_{tot}> = ½\, P_{tot}$ is known as the *root-mean-square*, or *rms* total power.

<u>Exercises:</u>

1. On a sheet of graph paper, draw an arbitrarily-shaped waveform, $y(x, t) = f(x - v_x t)$ representing the transverse displacement, $y(x, t=0)$ of a right-moving traveling wave along the x-axis at time $t = 0$ - *e.g.* a triangular-shaped pulse. Having chosen the amplitude, y_o and longitudinal velocity, v_x of this traveling wave, draw where this waveform is *e.g*. when $t = 1$ second, $t = 5$ seconds, and/or $t = 10$ seconds. Verify that $(x - v_x t)$ = constant = K for a given point on the waveform for each choice of time, t.

2. On a sheet of graph paper, draw an arbitrarily-shaped waveform, $y(x, t) = f(x + v_x t)$ representing the transverse displacement, $y(x, t=0)$ of a left-moving traveling wave along the x-axis at time $t = 0$ - *e.g.* a triangular-shaped pulse. Having chosen the amplitude, y_o and longitudinal velocity, $-v_x$ of this traveling wave, draw where this waveform is *e.g*. when $t = 1$ second, $t = 5$ seconds, and/or $t = 10$ seconds. Verify that $(x + v_x t)$ = constant = K' for a given point on the waveform for each choice of time, t.