transverse velocity - the total energy of the string is all in the form of potential energy. And on and on, through each successive cycle.

The time-average values of the kinetic energy and/or potential energy of the standing wave associated with the n^th normal mode of vibration of the entire string, of length, L are

$$\langle KE_n(t)\rangle = \tfrac{1}{2} KE_n(t) = \tfrac{1}{8} m\omega_n^2 C_n^2$$

and

$$\langle PE_n(t)\rangle = \tfrac{1}{2} PE_n(t) = \tfrac{1}{8} m\omega_n^2 C_n^2 = \langle KE_n(t)\rangle$$

This is because the time-averaged values of $\sin^2(\omega_n t)$ and $\cos^2(\omega_n t)$ are (averaging over one cycle, of period $\tau_n = 1/f_n$):

$$\langle \sin^2(\omega_n t)\rangle = \tfrac{1}{\tau_n}\int_{t=0}^{t=\tau_n} dt \sin^2(\omega_n t) = \tfrac{1}{2}$$

and

$$\langle \cos^2(\omega_n t)\rangle = \tfrac{1}{\tau_n}\int_{t=0}^{t=\tau_n} dt \cos^2(\omega_n t) = \tfrac{1}{2} = \langle \sin^2(\omega_n t)\rangle$$

If there are many normal modes of oscillation present as standing waves on a string, then the total energy, E_{tot} summing over all modes is given by:

$$E_{tot} = \sum_{n=1}^{n=\infty} E_n = \sum_{n=1}^{n=\infty} E_n(t) = \sum_{n=1}^{n=\infty} KE_n(t) + PE_n(t) = \tfrac{1}{4} m \sum_{n=1}^{n=\infty} \omega_n^2 C_n^2$$

The time-averaged total energy, $\langle E_{tot}\rangle$ summing over all modes is given by:

$$\langle E_{tot}\rangle = \sum_{n=1}^{n=\infty} \langle E_n\rangle = \sum_{n=1}^{n=\infty} \langle E_n(t)\rangle \equiv \sum_{n=1}^{n=\infty} \langle KE_n(t)\rangle + \langle PE_n(t)\rangle = \tfrac{1}{8} m \sum_{n=1}^{n=\infty} \omega_n^2 C_n^2 = \tfrac{1}{2} E_{tot}$$

Note that the time-averaged total energy, $\langle E_{tot}\rangle$ is *not* the same as the instantaneous energy, E_{tot} because of how it is defined - the time averages of the kinetic and potential energy for each mode, $\langle KE_n(t)\rangle$ and $\langle PE_n(t)\rangle$ are carried out *separately*. Thus, E_{tot} is also known as the *peak* total energy, while $\langle E_{tot}\rangle = ½\, E_{tot}$ is known as the so-called *root-mean-square*, or *rms* total energy.

Wave Power

The instantaneous power, $P_n(t)$ (in *Watts*, or *Joules*/*sec*) associated with the n^{th} normal mode of vibration of the entire string, of length, *L* at a given time, *t* in a standing wave is given by

$$P_n(t) = \frac{1}{\tau_n}\left[KE_n(t) + PE_n(t)\right] = \frac{1}{\tau_n} E_n(t) = \frac{1}{\tau_n} E_n = \frac{1}{\tau_n}\left[\tfrac{1}{4} m\omega_n^2 C_n^2\right] = P_n$$

Note that the instantaneous power has *no* time dependence - it is a constant, independent of time. The instantaneous power is also known as the *peak* power.