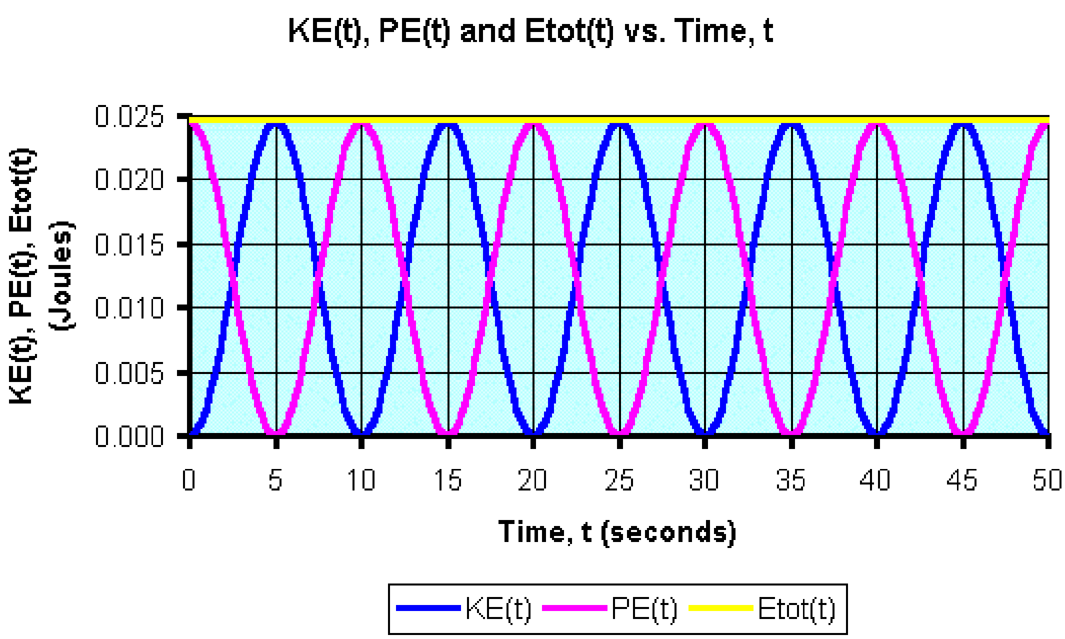
Note that the last term on the right hand side of the above equation used the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$. Thus, we discover that the total energy, $E_n(t)$ of the standing wave associated with the n^{th} normal mode of vibration of the entire string, of length, L as a function of time, t is a ***constant*** - the total energy, $E_n(t) = E_n$, *i.e*. it has no time dependence! Note also that the potential energy, kinetic energy, and total energy of the standing wave for the n^{th} normal mode of vibration all vary as the square of the frequency, since $\omega_n = 2\pi f_n$, and as the square of the amplitude, C_n, and depend linearly on the total mass, m of the string of length, L.

The potential energy, $PE_n(t)$ kinetic energy $KE_n(t)$ and total energy $E_n(t)$ associated with the n^{th} normal mode of vibration of the entire string, of length, L as a function of time, t are shown in the figure below. The values used for making this plot were $m = 1.0$ *kg* ($L = 10$ *m*), $n = 1$, $\tau_1 = 10.0$ *sec*, $f_1 = 1/\tau_1 = 1/20$ *Hz*, $|v_x| = 1.0$ *m/s* ($\lambda_1 = 2L = 20$ *m*), and $C_1 = 1.0$ *m*.



At one instant in time, say at $t = 0$ sec in the above figure, when the transverse displacement of the string is a maximum, with zero transverse velocity - the total energy of the string is all in the form of potential energy. One-quarter of a cycle later at $t = 5$ *sec*, when the string has maximum transverse velocity, the transverse displacement, $y_n(x,t=5) = 0$ = the equilibrium position of the string, and here the total energy of the string is all in the form of kinetic energy. Another one-quarter of a cycle later, at $t = 10$ sec, the transverse displacement of the string is again at a maximum (but is now on the *opposite* side of the equilibrium position, $y_n(x,t=5) = 0$), with zero transverse velocity, the total energy of the string is again all in the form of potential energy. One further quarter of a cycle later, at $t = 15$ *sec*, the string has again zero transverse displacement, $y_n(x,t=15) = 0$, but again has maximal transverse velocity, and again the total energy of the string is all in the form of kinetic energy. A further quarter of a cycle, at $t = 20$ sec, the string is back where it originally started, with maximum transverse displacement, again with zero