Mathematically, from calculus, this is stated as the derivative of $y(x, t)$ with respect to time, t - *i.e.* the time-rate of change of the transverse displacement, $y(x,t)$. The transverse velocity, $u_y(x, t)$ at the point x at time t is defined as:

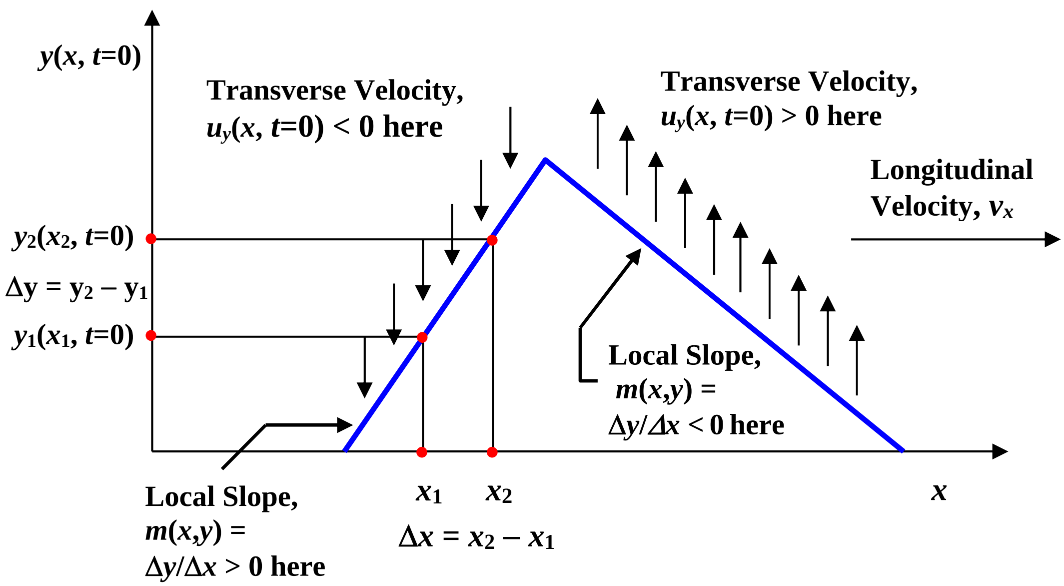
$$u_y(x,t) \equiv \frac{dy(x,t)}{dt}$$

However, again from calculus, by the *chain-rule of differentiation*, we can write $dy(x,t)/dt = df(x - v_x t)/dt$, thus:

$$u_y(x,t) \equiv \frac{df(x-v_x t)}{dt} = \frac{df(x-v_x t)}{d(x-v_x t)} * \frac{d(x-v_x t)}{dt} = \frac{\partial y(x,t)}{\partial x} * (-v_x) = -v_x \frac{\partial y(x,t)}{\partial x}$$

Here, we used the fact that the *total* derivative, $df(x - v_x t)/d(x - v_x t)$ = *partial* derivative = $\partial f(x - v_x t)/\partial(x - v_x t) = \partial y(x, t)/\partial x$ for *time t held fixed* (*i.e.* a constant), since the *longitudinal* velocity of the wave, v_x is a constant. The *longitudinal* velocity of the wave, v_x is defined as the *–ve* of the time-rate of change of the right-moving waveform, *i.e.* $v_x \equiv -d(x - v_x t)/\mathrm{dt}$. Note that the *local slope* of the string, $m(x, t) = \partial y(x, t)/\partial x \cong \Delta y(x, t)/\Delta x$ = change in y per change in x, at the point, x at a given time, t.

Thus, we see that the *transverse* velocity of the string, $u_y(x, t)$ at the point x and time t is the *product* of (the *local slope* of the string, $m(x, t) = \partial y(x, t)/\partial x$ at the point, x at *that* time, t) *and* (the negative of the *longitudinal* velocity of the transverse wave, $-v_x$). Note that the *partial* derivative of $y(x, t)$ with respect to x, $\partial y(x, t)/\partial x$ means differentiating $y(x, t)$ with respect to x (*i.e.* the change in y per change in x = "rise" over "run") *while holding the time, t fixed*. Thus, the (local) slope of the string, $m(x, t) = \partial y(x, t)/\partial x$ at the point, x is a *snapshot* of the string at the time, t. A right-moving, traveling *triangle* wave is shown in the figure below, which illustrates these concepts.



Note that by convention, a right-moving (left-moving) traveling wave has *longitudinal* velocity, $v_x > 0$ ($v_x < 0$), respectively.