where C_n is the amplitude of the n^{th} normal mode of vibration, and δ_n ($\delta_n{}' = \delta_n + 90^o$) is its phase, respectively.

Wave Energy

As mentioned earlier, a wave e.g. on a string has associated with it energy, E and momentum, P. If we consider an infinitesimally small segment of string, of equilibrium length, Δx, then for small-amplitude vibrations, the mass associated with this string segment is $\Delta m = \mu \Delta x$. The transverse velocity of this string segment is $u_y(x,t)$. The *kinetic* energy (in *Joules*) associated with this string segment, located at the point x along the string and at time t is given by:

$$\Delta KE(x,t) = \tfrac{1}{2}\Delta p_y^2(x,t)\big/\Delta m = \tfrac{1}{2}\Delta m\ u_y^2(x,t) = \tfrac{1}{2}\ \mu\ \Delta x\ \ u_y^2(x,t)$$

Transverse momentum and transverse velocity of a given string segment are related to each other by $\Delta p_y(x,t) = \Delta m\ u_y(x,t)$. The kinetic energy, $\Delta KE(x,t)$ associated with a vibrating string segment is a maximum when the magnitude of the transverse velocity, $|u_y(x,t)|$ of the string segment is a maximum. For sinusoidal-type transverse waves on a string, this occurs when the transverse displacement of the string segment, $y(x,t) = 0$.

We can obtain the total kinetic energy associated with the entire vibrating string at a given instant in time, t by summing up (*i.e.* adding) all of the individual kinetic energy contributions associated with each infinitesimal string segment, over the entire length of the string:

$$KE(t) = \sum_{n=1}^{N} \Delta KE(x,t) = \tfrac{1}{2}\mu \sum_{n=1}^{N} \Delta x\ \ u_y^2(x,t)$$

In the limit that the lengths of individual string segments, Δx become truly infinitesimal, then $\Delta x \to dx$, and the above summation formally becomes an integration over the length of the string, say from $x = 0$ to $x = L$:

$$KE(t) = \tfrac{1}{2}\mu \int_{x=0}^{x=L} dx\ u_y^2(x,t) = \frac{p_y^2(t)}{2m}$$

The *potential* energy, $\Delta PE(x,t)$ (in *Joules*) of an infinitesimal string segment of length, Δx of the vibrating string is associated with the (elastic) *stretching* of the string, under tension, T from its equilibrium position, $y(x,t) = 0$, to its transversely displaced position, $y(x,t) \neq 0$. (At the atomic level, the macroscopic stretching of the string corresponds to atoms making up the string material stretching apart from each other by very small distances, a fraction of an Angstrom ($1\ \AA = 10^{-10}\ m$). Typical inter-atomic separation distances are on the order of ~ few Angstroms.) The displacement amplitude is again assumed to be very small, such that although the string stretches elastically, the string tension, T remains constant. Then the potential energy, $\Delta PE(x,t) = T\Delta L$, where ΔL is the *elongation* (*i.e.* stretching) of the infinitesimal string segment, as shown in the figure below: