Note that the sequence of (11) time snapshots progress vertically downward in 1 second steps, starting at t = 0 sec, and ending at t = 10 sec. A full cycle of oscillation of this string has a period,  $\tau = 1/f = 20$  sec.



Y<sub>1</sub>(x, t) vs. x for Standing Wave n = 1 Fundamental

In general, it is entirely possible that *many* different normal modes of vibration could be simultaneously present on the string. In playing a guitar, this is in fact what actually happens! (We will discuss this in much greater detail in the lecture notes on Fourier analysis of waveforms.) The principle of linear superposition then tells us that the general solution of a vibrating string with fixed ends, with all possible normal modes of vibration present will have an overall transverse displacement that can be written as a sum (*i.e.* a linear superposition) of the individual transverse displacements,  $y_n(x, t)$  of these normal modes:

$$y_{tot}(x,t) = \sum_{n=1}^{n=\infty} y_n(x,t) = \sum_{n=1}^{n=\infty} [A_n \cos(\omega_n t) + B_n \sin(\omega_n t)] \sin(k_n x)$$

Thus, we can alternatively write the general solution,  $y_{tot}(x, t)$  as:

$$y_{tot}(x,t) = \sum_{n=1}^{n=\infty} y_n(x,t) = \sum_{n=1}^{n=\infty} C_n \cos(\omega_n t + \delta_n) \sin(k_n x) = \sum_{n=1}^{n=\infty} C_n \sin(\omega_n t + \delta_n') \sin(k_n x)$$
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