so-called $n = 1$ *fundamental*, with frequency, $f_1 = |v_x| / 2L$, and wavelength, $\lambda_1 = 2L$ is also known as the *first harmonic*. The 2nd harmonic, with $n = 2$, has frequency, $f_2 = |v_x| / L = 2f_1$ and wavelength, $\lambda_2 = L = \lambda_1/2$, and so on, for the higher harmonics, $n = 3, 4, 5, ...$ *etc*.

Thus, we find that the taught string, of length L with fixed ends at $x = 0$ and $x = L$ has *normal modes of vibration*, which for *each* such normal mode, the transverse displacement for an arbitrary point, x on the string, at *any* time t, is given by:

$$y_n(x,t) = [A_n \cos(\omega_n t) + B_n \sin(\omega_n t)] \sin(k_n x)$$

To connect this formula with the above formula for $y_{tot}(x, t)$, note that for a given normal mode of vibration, with harmonic number, $n = 1, 2, 3, 4, ...$ *etc*. we have simply re-defined the amplitudes, such that $A_n = 2A$ and $B_n = 2B$.

Note also that the physical amplitude of the n^{th} normal mode of vibration is $C_n = [A_n^2 + B_n^2]^{1/2}$ (*n.b.* this result follows from use of mathematics associated with *complex variables* - discussed in detail in the lecture notes on Fourier analysis (*aka* harmonic analysis) of waveforms). Thus, we can alternatively write the transverse displacement, $y_n(x, t)$ as:

$$y_n(x,t) = C_n \cos(\omega_n t + \delta_n) \sin(k_n x) = C_n \sin(\omega_n t + \delta_n') \sin(k_n x)$$

where C_n is the amplitude of the n^{th} normal mode of vibration, and δ_n ($\delta_n' = \delta_n + 90^o$) is its phase, respectively. Note that by a suitable re-definition of the zero of time, *e.g.* $\omega_n t' = \omega_n t + \delta_n$, or $t' = t + \delta_n/\omega_n$, or, instead, $\omega_n t^* = \omega_n t + \delta_n'$, or $t^* = t + \delta_n'/\omega_n$, the above expression for the transverse displacement, $y_n(x, t)$ for the n^{th} normal mode of vibration can be rewritten either as:

$$y_n(x,t) = C_n \cos(\omega_n t') \sin(k_n x)$$

or as:

$$y_n(x,t) = C_n \sin(\omega_n t^*) \sin(k_n x)$$

The above mathematical expression(s) for the transverse displacement, $y_n(x, t)$ associated with the n^{th} normal mode of vibration of a string of length, L with fixed end-points at $x = 0$ and $x = L$ describe a transverse *standing wave*. A standing wave is a wave that is *not* a *traveling* wave - it has no *longitudinal* motion, because, from the above mathematics, we see that a standing wave is in fact nothing more than a linear superposition of appropriately-phased right- and left-moving traveling waves of equal amplitude and frequency! The *envelope* of the standing wave as a function of the position x, over the interval $0 \le x \le L$ along the length of the string is given by $C_n \sin(k_n x)$. The envelope of the standing wave is thus a snapshot of the standing wave, *e.g.* at $t' = 0$.

The following plot shows a snapshot time sequence of one half of a cycle of oscillation of a transverse standing wave - the $n = 1$ fundamental mode of vibration on a string of length, $L = 10$ *m*, with fixed ends located at $x = 0$ and $x = L = 10$ *m*. The amplitude of the pulse is $A_1 = 1.0$ *m*, the wavelength of the fundamental is $\lambda = 2L = 20$ *m*, the longitudinal wave speed $|v_x| = 1.0$ *m/sec*, thus the frequency is $f = |v_x|/\lambda = 0.05$ *Hz*.