The boundary condition, $y_{\text{tot}}(x{=}L, t) = 0$ for the fixed end at $x = L$, which *must* be obeyed for *any* time, t, can *only* be satisfied if

$$y_{tot}(x=L,t) = A\sin(kL-\omega t) + B\cos(kL-\omega t) + C\sin(kL+\omega t) + D\cos(kL+\omega t) = 0$$

However, because of the requirement from the boundary condition $y_{\text{tot}}(x{=}0, t) = 0$, that the amplitudes $C = A$ *.and.* $B = -D$, this can be rewritten as:

$$y_{tot}(x=L) = A[\sin(kL-\omega t) + \sin(kL+\omega t)] + B[\cos(kL-\omega t) - \cos(kL+\omega t)]$$

Using the angle-addition formulae:

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

and

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

then:

$$\begin{aligned} y_{tot}(x=L,t) = {} & A[\sin(kL)\cos(\omega t) - \cos(kL)\sin(\omega t) + \sin(kL)\cos(\omega t) + \cos(kL)\sin(\omega t)] \\ & + B[\cos(kL)\cos(\omega t) + \sin(kL)\sin(\omega t) - \cos(kL)\cos(\omega t) + \sin(kL)\sin(\omega t)] \end{aligned}$$

which becomes:

$$\begin{aligned} y_{tot}(x=L,t) &= 2A\sin(kL)\cos(\omega t) + 2B\sin(kL)\sin(\omega t) \\ &= 2[A\cos(\omega t) + B\sin(\omega t)]\sin(kL) \end{aligned}$$

The *only* way that the boundary condition, $y_{\text{tot}}(x{=}L, t) = 0$ for the fixed end at $x = L$ can be satisfied for *any* time, t is if $\sin(kL) = 0$. This happens when $kL = n\pi$, where n is a positive integer, *i.e.* $n = 1, 2, 3, 4, \ldots$ *etc*. Then $\sin(n\pi) = 0$, and hence $y_{\text{tot}}(x{=}L, t) = 0$ is satisfied for any time t. Since $k = 2\pi/\lambda$, the case $n = 0$ is not allowed, since the distance between end supports, L is finite, and $n = 0$ corresponds to $\lambda = \infty$. Physically, the requirement that the transverse displacement at $x = L$ be zero is the same requirement as that for the transverse displacement at $x = 0$.

Note that the relation $kL = 2\pi L/\lambda = n\pi$ implies that $\lambda = 2L/n$ - *i.e.* that the allowed wavelengths associated with transverse waves propagating on this string of length L between the fixed endpoints $x = 0$ and $x = L$, can only be integer fractions of the total length, L of the string! We can denote these special wavelengths, λ and wavenumbers, k by the subscript n, i.e. $\lambda_n = 2L/n$ and $k_n = n\pi/L$, $n = 1, 2, 3, 4, \ldots$ *etc*. Since the longitudinal wave speed, $|v_x| = f\lambda = \omega/k$, then $f = |v_x|/\lambda_n = n|v_x|/2L$; we can denote these special frequencies, f and angular frequencies, $\omega = 2\pi f$ as $f_n = n|v_x|/2L$ and $\omega_n = n\pi|v_x|/L$, $n = 1, 2, 3, 4, \ldots$ *etc*.

Thus, we discover that only *certain* wavelengths, corresponding to *certain* frequencies of sinusoidal-type transverse waves are able to propagate on a taught string of length, L, with fixed ends! These frequencies, $f_n = n|v_x|/2L$ (wavelengths, $\lambda_n = 2L/n$) with $n = 1, 2, 3, 4, \ldots$ *etc*. are integer multiples (integer fractions), or *harmonics*, of the *fundamental* frequency (*fundamental* wavelength), $f_1 = |v_x|/2L$ ($\lambda_1 = 2L$), respectively. Note that the