10^{25}–10^{30} atoms, depending on the mass of the vibrating object) in which the wave is propagating are also vibrating (scattering), but with extremely small amplitudes. However, adding up all of the individual atomic contributions results in the macroscopic vibrations that we can see and hear with our own eyes and ears, respectively!

Standing Waves

We now consider wave propagation on a taught string, of length, L, tension T and mass per unit length, μ, with fixed end supports at $x = 0$ and $x = L$. If we consider sinusoidal-type transverse traveling wave solutions of the wave equation, mathematically, the most general possible solution of the wave equation for waves propagating on such a string consists of a linear combination of both right- and left-moving sine and cosine-type traveling waves - four total:

$$y_{tot}(x,t) = A\sin(kx - \omega t) + B\cos(kx - \omega t) + C\sin(kx + \omega t) + D\cos(kx + \omega t)$$

Here, A, B, C, D are the amplitudes (in *meters*, m) of the four types of sinusoidal waves, the wavenumber, $k = 2\pi/\lambda$ (in units of $1/m$, or m^{-1}), where λ is the wavelength (*meters*), associated with the frequency, f (*Hz*) and angular frequency $\omega = 2\pi f$ (*radians*/*sec*). The longitudinal wave speed on the string is $|v_x| = f\lambda = (T/\mu)^{1/2}$ (*m*/*sec*).

First, we note that the sine and cosine functions, sin(x) and cos(x) have some useful (and important) <u>*reflection symmetry*</u> properties - namely that $\sin(-x) = -\sin(x)$, and that $\cos(-x) = +\cos(x)$. When we change (*i.e.* reflect) $x \to -x$, the $\sin(x)$ function changes to $\sin(-x)$, which changes sign, to $-\sin(x)$ under this reflection operation (by the "*all-sin-tan-cos*" rule - the nemonic for which trigonometrical function is positive in each of the four quadrants associated with $0^o \le x \le 360^o$) . Thus, we say that the $\sin(x)$ function has <u>*odd*</u> reflection symmetry (because of the sign change of $\sin(-x) = -\sin(x)$). Because the $\cos(x)$ function does not change sign under this reflection operation, we say that the $\cos(x)$ function has <u>*even*</u> reflection symmetry (because $\cos(-x) = +\cos(x)$).

The boundary condition, $y_{tot}(x=0, t) = 0$ for the fixed end at $x = 0$, which *must* be obeyed for *any* time, t, can *only* be satisfied if

$$y_{tot}(x=0,t) = -A\sin(\omega t) + B\cos(\omega t) + C\sin(\omega t) + D\cos(\omega t) = 0$$

We group like terms (sines and cosines) together:

$$y_{tot}(x=0,t) = (C - A)\sin(\omega t) + (B + D)\cos(\omega t) = 0$$

The *only* way that the boundary condition, $y_{tot}(x=0, t) = 0$ for the fixed end at $x = 0$ can be satisfied for *any* time, t is if the amplitudes, $C = A$ *.and.* $B = -D$. Physically, this means that the amplitudes of the right- and left-moving traveling sine-waves *must* be equal, and be in phase with each other. Physically, the amplitudes of the right- and left-moving traveling cosine-waves *must* also be equal, but be out-of-phase with each other by 180^o (use the angle-addition formula to show that $\cos(x + 180^o) = -\cos(x)$).