partial derivative, $\partial y(x,t)/\partial x$, then *evaluate* it at $x = L$. Physically, the requirement that the slope, $\partial y/\partial x = 0$ for reflection of waves at a free-end of a taught string simply arises because of the fact that no *transverse* force, F_y can exist at this kind of end point.

If we have a right-moving transverse traveling wave, again in the shape of a gaussian pulse, $y(x,t) = y_o \exp\{-(x-v_x t)^2\}$, then as this waveform reaches the fixed endpoint at $x = L$, the free-end boundary condition $\partial y(x=L,t)/\partial x = 0$ must be obeyed. As the pulse impinges on the free end support, the only way that $\partial y(x=L,t)/\partial x = 0$ can be obeyed is if this pulse is simultaneously reflected, but with the *same* polarity as the incident wave. Again, reflection means that the right-moving gaussian-shaped transverse traveling wave, $y(x,t) = y_o \exp\{-(x-v_x t)^2\}$ is converted into a left-moving gaussian-shaped transverse traveling wave, $y(x,t) = y_o \exp\{-[(x-2L)+v_x t]^2\}$at the point $x = L$; maintaining the original polarity means that the now left-moving gaussian-shaped transverse traveling wave has the same phase as the original wave on the y-axis, i.e. the amplitude y_o remains y_o. Thus, the gaussian-shaped transverse traveling wave reflected from a free end at $x = L$ is of the form $y(x,t) = y_o \exp\{-[(x-2L)+v_x t)^2$, after reflection.

Again, an equivalent way to think about this process is to imagine a special kind of mirror at the free endpoint $x = L$. As the right-moving gaussian-shaped transverse traveling wave, $y(x,t) = y_o \exp\{-(x-v_x t)^2\}$ approaches the free end, an observer can see this wave directly, but looking in the special mirror, this observer *also* sees a *left*-moving, gaussian-shaped transverse traveling wave, with the *same* polarity as the incident wave, $y(x,t) = y_o \exp\{-[(x-2L)+v_x t]^2\}$ approaching the endpoint at $x = L$, but from *behind* the mirror. Again, this left-moving, same-polarity pulse, approaching from *behind* the end point is also known as an *image pulse*. The free-end boundary condition, $\partial y_{tot}(x=L,t)/\partial x = 0$ for the incident, right-moving transverse traveling wave and its image pulse, independent of the detailed shape (transverse profiles) of these transverse traveling waves, is generically given by:

$$\partial y_{tot}(x=L,t)/\partial x = \partial f(x-v_x t)/\partial x|_{x=L} + \partial f(x+v_x t)/\partial x|_{x=L} = 0$$

Where "$|_{x=L}$" means evaluating the partial derivatives, $\partial/\partial x$ of these functions at $x = L$. Thus, the free-end boundary condition, $\partial y_{tot}(x=L,t)/\partial x = 0$ can only be satisfied if $\partial f(x-v_x t)/\partial x|_{x=L} = -\partial f(x+v_x t)/\partial x|_{x=L}$. If we *integrate* this relation, i.e.

$$\int \frac{\partial f\,(x - v_x t)}{\partial x}\Big|_{x=L}\, dx = -\int \frac{\partial f\,(x + v_x t)}{\partial x}\Big|_{x=L}\, dx$$

we obtain the result:

$$f(x-v_x t)|_{x=L} = +\, f(x+v_x t)|_{x=L}$$

The following plot shows a snapshot time sequence of a right-moving gaussian-shaped transverse traveling wave reflecting from a free end, located at $x = L = 10\ m$. The amplitude of the pulse is $y_o = 0.5\ m$, the longitudinal wave speed $v_x = 1.0\ m/sec$. Note that the sequence of (11) time snapshots progress vertically downward in 1 second steps, starting at t = 5 *sec*, and ending at t = 15 *sec*. The left-moving image pulse is used to represent the reflection in the physical region ($x \le L = 10\ m$).