gaussian-shaped transverse traveling wave is "flipped over" on the $y$-axis, *i.e.* $y_o$ changes to $-y_o$. This is equivalent to a phase change, or phase shift of the reflected wave relative to the incident wave of 180°. Thus, the gaussian-shaped transverse traveling wave reflected from a fixed end located at $x = L$ is of the form $y(x,t) = -y_o \exp\{-[(x-2L)+v_x t)^2$, after reflection.

Another equivalent way to think about this process is to imagine a special kind of mirror at the fixed endpoint $x = L$. As the right-moving gaussian-shaped transverse traveling wave, $y(x,t) = y_o \exp\{-(x-v_x t)^2\}$ approaches the fixed end, an observer can see this wave directly, but looking in the special mirror, this observer *also* sees a *left*-moving, but *polarity-inverted* gaussian-shaped transverse traveling wave, $y(x,t) = -y_o \exp\{-[(x-2L)+v_x t)^2$ approaching the endpoint at $x = L$, but from *behind* the mirror. This left-moving, polarity inverted pulse, approaching from *behind* the end point is known as an *image pulse*. This way of thinking about the problem arises from use of the principle of linear superposition. The fixed-end boundary condition, $y_{tot}(x=L,t) = 0$ for the incident, right-moving transverse traveling wave and its image pulse, independent of the detailed shape (transverse profiles) of these transverse traveling waves, is generically given by:

$$y_{tot}(x=L,t) = f(x-v_x t)|_{x=L} + f(x+v_x t)|_{x=L} = 0$$

where "$|_{x=L}$" means evaluating these functions at $x = L$. Thus, the fixed end boundary condition, $y_{tot}(x=L,t) = 0$ can only be satisfied if $f(x-v_x t)|_{x=L} = -f(x+v_x t)|_{x=L}$.

Note that the peak of the right-moving gaussian-shaped transverse traveling wave arrives at $x = L$ when $t = L/v_x$. The peak location, $x$ of the right-moving gaussian pulse occurs at a given time, $t$ when $(x-v_x t) = 0$. Thus, at $t = 0$, $x = 0$. Note also that the left-moving gaussian-shaped transverse traveling wave has had its origin shifted, such that when $t = 0$, its gaussian peak occurs at $x = 2L$, and when $t = L/v_x$, its gaussian peak also is at $x = L$. The peak, $x$ of this left-moving gaussian pulse occurs at a given time, $t$ when $[(x-2L)+v_x t] = 0$. Thus, at $t = 0$, $x = 2L$ for the left-moving gaussian pulse.

As time progresses, the two pulses move closer and closer to each other, *passing through* each other at $x = L$, and then keep on going. During the time that the two pulses overlap each other, the overall transverse displacement of the string at $x = L$ is just the *linear superposition* of the two waves, *i.e.*

$$y_{tot}(x=L,\, t) = y_{right}(x=L,\, t) + y_{left}(x=L,\, t) = y_o \exp\{-(L-v_x t)^2\} - y_o \exp\{-(-L+v_x t)^2\}$$

In general, for *any* point $x$, and time $t$, we will have, for reflection at a fixed end:

$$y_{tot}(x,\, t) = y_{right}(x,\, t) + y_{left}(x,\, t) = y_o \exp\{-(x-v_x t)^2\} - y_o \exp\{-[(x-2L)+v_x t]^2\}$$

Of course, what we see physically is only the portion of the string for the two waves with $x < L$. Mathematically, we can represent the wave that is reflected from a fixed end as an image wave, traveling in the opposite direction, and with inverted polarity, to the incoming traveling wave incident on the fixed end. Thus, because it takes a finite time for such a wave to undergo reflection from a fixed end, we can represent this process as a linear superposition of two waves, the original, right-moving transverse traveling wave,