This is a very nice signal - it is a DC signal proportional to the signal amplitude.

Narrow band detection

Now suppose the input is made up of signal plus noise. The PSD and low pass filter only detect signals whose frequencies are very close to the lock-in reference frequency. Noise signals at frequencies far from the reference are attenuated at the PSD output by the low pass filter (neither ω_{noise}-ω_{ref} nor ω_{noise}+ω_{ref} are close to DC). Noise at frequencies very close to the reference frequency will result in very low frequency AC outputs from the PSD ($|\omega_{noise}-\omega_{ref}|$ is small). Their attenuation depends upon the low pass filter bandwidth and roll-off. A narrower bandwidth will remove noise sources very close to the reference frequency, a wider bandwidth allows these signals to pass. The low pass filter bandwidth determines the bandwidth of detection. Only the signal at the reference frequency will result in a true DC output and be unaffected by the low pass filter. This is the signal we want to measure.

Where does the lock-in reference come from?

We need to make the lock-in reference the same as the signal frequency, i.e. $\omega_r = \omega_L$. Not only do the frequencies have to be the same, the phase between the signals can not change with time, otherwise $\cos(\theta_{sig} - \theta_{ref})$ will change and V_{psd} will not be a DC signal. In other words, the lock-in reference needs to be phase-locked to the signal reference.

Lock-in amplifiers use a phase-locked-loop (PLL) to generate the reference signal. An external reference signal (in this case, the reference square wave) is provided to the lock-in. The PLL in the lock-in locks the internal reference oscillator to this external reference, resulting in a reference sine wave at ω_r with a fixed phase shift of θ_{ref}. Since the PLL actively tracks the external reference, changes in the external reference frequency do not affect the measurement.

All lock-in measurements require a reference signal.

In this case, the reference is provided by the excitation source (the function generator). This is called an external reference source. In many situations, the SR830's internal oscillator may be used instead. The internal oscillator is just like a function generator (with variable sine output and a TTL sync) which is always phase-locked to the reference oscillator.

Magnitude and phase

Remember that the PSD output is proportional to $V_{sig}\cos\theta$ where $\theta = (\theta_{sig} - \theta_{ref})$. θ is the phase difference between the signal and the lock-in reference oscillator. By adjusting θ_{ref} we can make θ equal to zero, in which case we can measure V_{sig} ($\cos\theta$=1). Conversely, if θ is 90°, there will be no output at all. A lock-in with a single PSD is called a single-phase lock-in and its output is $V_{sig}\cos\theta$.

This phase dependency can be eliminated by adding a second PSD. If the second PSD multiplies the signal with the reference oscillator shifted by 90°, i.e. $V_L\sin(\omega_L t + \theta_{ref} + 90°)$, its low pass filtered output will be

$$V_{psd2} = 1/2\ V_{sig}V_L\sin(\theta_{sig} - \theta_{ref})$$

$$V_{psd2} \sim V_{sig}\sin\theta$$

Now we have two outputs, one proportional to $\cos\theta$ and the other proportional to $\sin\theta$. If we call the first output X and the second Y,

$$X = V_{sig}\cos\theta \qquad Y = V_{sig}\sin\theta$$

these two quantities represent the signal as a vector relative to the lock-in reference oscillator. X is called the 'in-phase' component and Y the 'quadrature' component. This is because when θ=0, X measures the signal while Y is zero.

By computing the magnitude (R) of the signal vector, the phase dependency is removed.

$$R = (X^2 + Y^2)^{1/2} = V_{sig}$$

R measures the signal amplitude and does not depend upon the phase between the signal and lock-in reference.

A dual-phase lock-in, such as the SR830, has two PSD's, with reference oscillators 90° apart, and can measure X, Y and R directly. In addition, the phase θ between the signal and lock-in reference, can be measured according to

$$\theta = \tan^{-1}(Y/X)$$