G. The Tempered Scale:

Make all intervals the same for the 12 notes. Create an interpolated musical scale, known as the tempered scale with 12 equally-spaced semitones for the 12 notes of this musical scale – then divide up the octave into 1200 cents. Thus, 100 cents = 1 tempered semitone, and one octave = 12 tempered semitones. Not a "perfect" musical scale by any means, but the main advantage is can then play a given piece of music in any key, without it sounding "worse" in one key than another.

We can mathematically define the semitone in the tempered scale as: $\boldsymbol{a} = (2)^{1/12} = 1.05946$. Then one cent, $1¢ = (2)^{1/1200} = 1.000577$ in the tempered scale. Each note is built up from previous note by adding one power of the semitone, $\boldsymbol{a}$, as shown in the figure below:

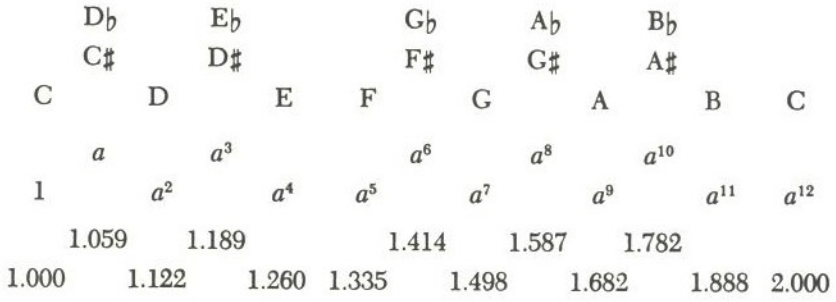

FIG. 10. The tempered scale.

The 2nd note D is obtained from C by a^2, the twelfth (*i.e.* octave) high-C note is obtained from the low-C by a factor of a^{12}, etc.

H.) Location of Frets on the Fretboard of a Guitar (Tempered Scale):

For the tempered semitone, $\boldsymbol{a} = (2)^{1/12} = 1.05946$, then for a scale length $\boldsymbol{L}$:

For the first 12 frets on the fretboard (first octave), where $\boldsymbol{n}$ = fret # (1-12):

$$Fret\ Location_{n-th\ fret} = L \Big/ 2^{\frac{n}{12}}$$

For frets 13-24 on the fretboard (2nd octave), where $\boldsymbol{n}$ = fret # (13-24):

$$Fret\ Location_{n-th\ fret} = \tfrac{1}{2} L \Big/ 2^{\frac{n}{12}}$$

For frets 25-36 on the fretboard (3rd octave), where $\boldsymbol{n}$ = fret # (25-36):

$$Fret\ Location_{n-th\ fret} = \tfrac{1}{4} L \Big/ 2^{\frac{n}{12}}$$