

Note that:

- The adjacent note pairs (E & F) and (B & C) have smaller musical intervals than (C & D), (D & E), (F & G), (G & A), and (A & B).
- Compare the interval (*i.e.* frequency ratio) of:
 $\frac{9}{8} = 1.125$, which is known as a **Pythagorean wholetone**,
to that of:
 $\frac{256}{243} = 1.053$, which is known as a **Pythagorean diatonic semitone**.
- The seven notes of the Pythagorean scale correspond to the seven white keys on the piano.

What about the black keys of the piano?

9. Go down a 4th from B. This is the note F[#], with frequency $\frac{3}{4} \bullet \frac{243}{128} f = \frac{729}{512} f$.

**The interval (frequency ratio): F[#] - G = $\frac{3/2}{729/512} = \frac{256}{243} = 1.053$,
which is a Pythagorean diatonic semitone.**

**The interval (frequency ratio): F - F[#] = $\frac{729/512}{4/3} = \frac{2187}{2048} = 1.068$,
which is known as a Pythagorean **chromatic** semitone.**

Thus, there are two different sizes of semitones:

**Pythagorean Wholetone = $\frac{9}{8} = 1.125$
Pythagorean Diatonic Semitone = $\frac{256}{243} = 1.0534979$
Pythagorean Chromatic Semitone = $\frac{2187}{2048} = 1.0678711$**

The Chromatic Semitone is slightly larger than the Diatonic Semitone!

**10. Go up a 5th from F[#]. This is the note B[#], with frequency $\frac{3}{2} \bullet \frac{729}{512} f = \frac{2187}{1024} f$.
 $> 2f$ (= high C)!**

This key doesn't exist on the piano! B[#] is the enharmonic equivalent to C, but it is not the note C in the Pythagorean musical scale!

We have more problems – start with F and use the circle of fifths to generate the notes of the black keys on a piano: F-C-G-D-A-E-B-F[#]-C[#]-G[#]-D[#]-A[#]-E[#]

The key E[#] doesn't exist on the piano! E[#] is the enharmonic equivalent to F, but it is not the note F in the Pythagorean musical scale! The frequency ratio of E[#]:F is $\{[\frac{3}{2}]^{12} \times [\frac{1}{2}]^7\}:1 = 531441:524288 = 1.01364..$ *i.e.* E[#] is higher than F by this amount!

This interval (frequency ratio) is also the same as that between the Pythagorean chromatic semitone and the Pythagorean diatonic semitone, *i.e.* $\frac{2187}{2048} / \frac{256}{243} = 1.01364$, which is known as the **Pythagorean comma.**