

A.) Pentatonic (5-Note) Scale:

This is the “simplest” musical scale. Use the octave, 4th and 5th as consonant intervals (as expressed by their respective frequency ratios) to build up the so-called pentatonic/5-note scale:

1. Start with the note C, with frequency f . This is the note low-C.
2. The octave is another C with frequency $2f$. This is the note high-C.
3. Go down a 5th from high-C (= going up a 4th from low-C).
This is the note F, with frequency $\frac{4}{3}f$.
4. Go up a 5th from low-C (= going down a 4th from high-C).
This is the note G, with frequency $\frac{3}{2}f$.
5. Go down a 4th from G (= going up a 5th from G and then down an octave).
This is the note D, with frequency $\frac{3}{4} \cdot \frac{3}{2}f = \frac{9}{8}f$.
6. Go up a 5th from D. This is the note A, with frequency $\frac{3}{2} \cdot \frac{9}{8}f = \frac{27}{16}f$.

The 5-Note Pentatonic Scale:

Note:	C	D	F	G	A	C
Frequency:	f	$\frac{9}{8}f$	$\frac{4}{3}f$	$\frac{3}{2}f$	$\frac{27}{16}f$	$2f$

FIG. 3. A pentatonic scale.

Relative Ratio (to fundamental):	1	$\frac{9}{8}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{27}{16}$	2
Interval (Frequency Ratio):		$\frac{9}{8}$	$\frac{32}{27}$	$\frac{9}{8}$	$\frac{9}{8}$	$\frac{32}{27}$

B.) Pythagorean (7-Note) Scale:

The pentatonic 5-note scale has 2 notes missing (E & B), so we continue:

7. Go down a 4th from A. This is the note E, with frequency $\frac{3}{4} \cdot \frac{27}{16}f = \frac{81}{64}f$.
8. Go up a 5th from E. This is the note B, with frequency $\frac{3}{2} \cdot \frac{81}{64}f = \frac{243}{128}f$.

The 7-Note Pythagorean Scale:

Note:	C	D	E	F	G	A	B	C
Frequency:	1	$\frac{9}{8}$	$\frac{81}{64}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{27}{16}$	$\frac{243}{128}$	2
Interval:		$\frac{9}{8}$	$\frac{9}{8}$	$\frac{256}{243}$	$\frac{9}{8}$	$\frac{9}{8}$	$\frac{9}{8}$	$\frac{256}{243}$

FIG. 4. The Pythagorean scale.