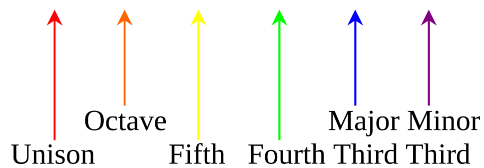
Thus, the ratio of the lengths of the two string segments R is also the {inverse} ratio of the two frequencies associated with the vibrating string segments on either side of the bridge:

$$R = \frac{x}{L-x} = \frac{v/2f_x}{v/2f_{L-x}} = \frac{f_{L-x}}{f_x}$$

Consonance occurs when the lengths (frequencies) of the two string segments are in very special/unique ***integer*** ratios, R (and/or $1/R$), respectively of:

$$R = \frac{x}{L-x} = \frac{f_x}{f_{L-x}} = 1:1,\ \ 1:2,\ \ 2:3,\ \ 3:4,\ \ 4:5,\ \ 5:6,\ \ ....$$

$$1/R = \frac{L-x}{x} = \frac{f_{L-x}}{f_x} = 1:1,\ \ 2:1,\ \ 3:2,\ \ 4:3,\ \ 5:4,\ \ 6:5,\ \ ....$$



These ***integer*** frequency ratios relate ***directly*** to two notes played in unison, octave, fifth, fourth, major/minor thirds and second of the ***just diatonic musical scale*** – (see below)!

Dissonance occurs when the length of string segments (*i.e*. frequency ratios) are ***far*** from/are ***not*** integers.

When two (or more) musical tones are consonant, the ***phase relation*** of the higher frequency relative to the lower frequency is ***time-independent***. The resulting overall waveform is ***stationary/time-stable***, with a ***repeat time*** of the waveform that is relatively short – min{m:n} where $1/R = m/n$ {see figure on next page}.

The phase-stability of the waveform for a consonant sound makes it particularly easy for the human ear/brain to recognize (analyze). Also, note that the harmonic(s) of the higher frequency tone – *e.g*. major 3rd or fifth, tend to line up/coincide with the harmonics of the lower frequency tone! (Quadratic) non-linear responses present in the human ear/brain generate/create sum & difference frequencies, $(\boldsymbol{f_{L-x}} + \boldsymbol{f_x})$ and $|\boldsymbol{f_{L-x}} - \boldsymbol{f_x}|$ that also perfectly/exactly line up with the harmonics of the two tones, and again which have a time-independent/stationary phase relation relative to the fundamental of the lowest tone! The human ear/brain thus perceives consonant tones as very special and unique!