

The human ear/brain is capable of perceiving a ***fundamental*** even when ***no*** fundamental is actually present!!! This is the so-called ***missing fundamental effect***.

This effect is {again} a consequence of the non-linear response in/inside the human ear itself, and/or a non-linear response(s) in the human brain's ***processing*** of frequency information – whenever *e.g.* a ***quadratic*** non-linear response exists (in any system), if two signals *A* and *B* with frequencies  $f_A$  and  $f_B$  are input to that system, then sum and difference frequencies ( $f_A + f_B$ ) and  $|f_A - f_B|$  are produced! Thus, *e.g.* a 2<sup>nd</sup> harmonic  $2f_I$  and a 3<sup>rd</sup> harmonic  $3f_I$  can produce a “missing” fundamental from the difference frequency,  $|3f_I - 2f_I| = f_I$  !!! For further details on distortion, read Physics 406 Lecture Notes on “Theory of Distortion I & II”.

For some musical instruments – *e.g.* the trumpet, the oboe and/or the bassoon – the 2<sup>nd</sup> (or even 3<sup>rd</sup> and higher) harmonics can actually have a ***larger*** amplitude than that of the fundamental, however we perceive/hear the “note” that is played on the trumpet (and/or oboe, bassoon) as that of the fundamental!!!

The harmonic spectra – *aka* power spectral density functions  $S_{pp}(f)$  vs.  $f$  and associated {time-averaged} relative phase harmonic phasor plots are shown below – *e.g.* for the steadily-played notes A4 (440.0 Hz) played on the oboe, and F2 (87.3 Hz) played on the bassoon:

