

$\cos(A - B) = \cos A \cos B + \sin A \sin B$ , hence we see that:

$$A_n(t) = A_n \cos \varphi_n \cos(n\omega_1 t) + A_n \sin \varphi_n \sin(n\omega_1 t) = A_n \cos(n\omega_1 t - \varphi_n)$$

We also see that:  $A_n = \sqrt{A_n^2 \cos^2 \varphi_n + A_n^2 \sin^2 \varphi_n} = \sqrt{a_n^2 + b_n^2}$  and that:  $\varphi_n = \tan^{-1}(b_n/a_n)$ .

Hence, we can equivalently write the Fourier series expression:

$$A_{tot}(t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_1 t)$$

as:

$$A_{tot}(t) = a_o + \sum_{n=1}^{\infty} A_n \cos(n\omega_1 t + \varphi_n)$$

with:  $A_n = \sqrt{a_n^2 + b_n^2}$  and:  $\varphi_n = \tan^{-1}(b_n/a_n)$ .

Fourier analysis applies to any/all kinds of complex periodic waveforms – electrical signals, optical waveforms, *etc.* - **any** periodic waveform (temporal, spatial, *etc.*). Please see/read Physics 406 Series of Lecture Notes on Fourier Analysis I-IV for much more details/info...