<u>Tone Structure</u>:

We can build up/construct a complex waveform by linear superposition/linear combination of the harmonics:

$$A_{tot}(t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_1 t)$$
$$= a_o + (a_1 \cos\omega_1 t + a_2 \cos 2\omega_1 t + a_3 \cos 3\omega_1 t + a_4 \cos 4\omega_1 t + \ldots)$$
$$+ (b_1 \sin\omega_1 t + b_2 \sin 2\omega_1 t + b_3 \sin 3\omega_1 t + b_4 \sin 4\omega_1 t + \ldots)$$

⇒ See/try out the UIUC P406's **Fourisim.exe** and/or **Guitar.exe** computer demo programs to learn/see/hear more about complex waveforms…

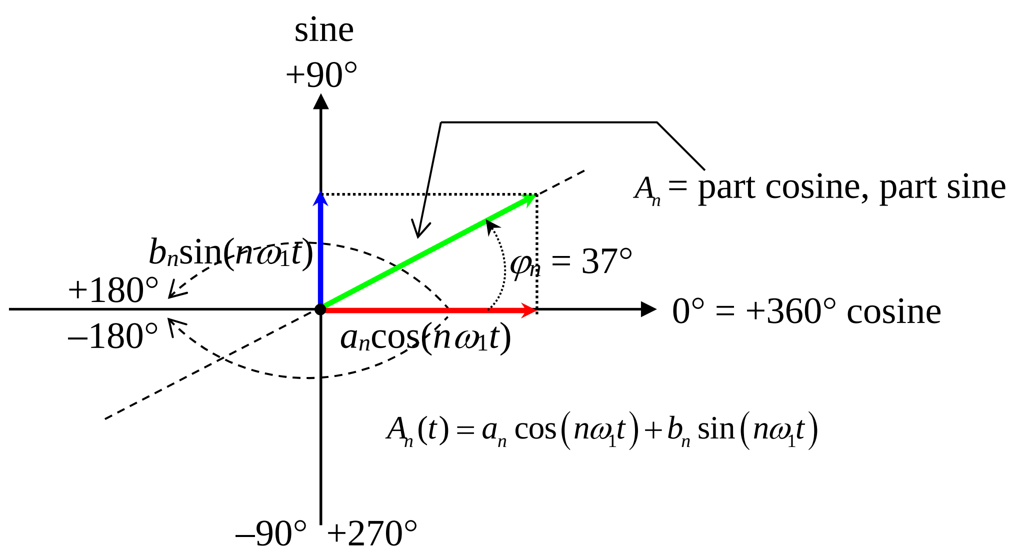
<u>Harmonic</u> <u>Synthesis</u>: ***<u>Adding</u>*** harmonics together to produce a complex waveform.

⇒Please see & hear the Hammond Organ harmonic synthesis demo…⇐

<u>Harmonic</u> <u>Analysis</u>: ***<u>Decomposing</u>*** a complex waveform into constituent harmonics.

<u>Any</u> complex periodic waveform can be analyzed into its constituent harmonics *i.e.* harmonic amplitudes and phases (*e.g.* relative to the fundamental).

Pure sine $\{b_n \sin(n\omega_1 t)\}$ and cosine $\{a_n \cos(n\omega_1 t)\}$ waves have a 90° phase relation with respect to each other, *e.g.* at a given time, t:



From the above phasor diagram, note that we can equivalently rewrite $A_n(t)$ as:

$$A_n(t) = a_n \cos(n\omega_1 t) + b_n \sin(n\omega_1 t) = A_n \cos(n\omega_1 t - \varphi_n)$$

From trigonometry, we see that: $a_n = A_n \cos\varphi_n$ and $b_n = A_n \sin\varphi_n$, and since: