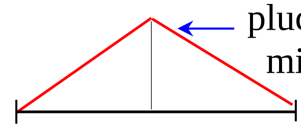
<u>Harmonics of the fundamental</u> also known as ***<u>partials</u>***

The fundamental = 1st harmonic/partial
The 2nd harmonic/partial has $f_2 = 2f_1$ (aka 1st <u>overtone</u>)
The 3rd harmonic/partial has $f_3 = 3f_1$ (aka 2nd <u>overtone</u>) …. *etc.*

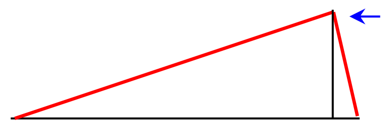
A vibrating string (guitar/violin/piano) contains many harmonics = complex tone.

The detailed ***<u>shape</u>*** of a plucked string on a guitar (or violin) uniquely determines its harmonic content! Please see/hear/touch Physics 406POM **Guitar.exe** demo!

"mellow" – less high harmonics



"bright" – more high harmonics



The geometrical ***<u>shape</u>*** of the string at the instant (t = 0) that the string is plucked **<u>defines</u>** the ***<u>amplitudes</u>*** (& ***<u>phases</u>***) of the harmonics associated with standing wave on the string:

<u>Transverse Displacement of String</u>: $y(x,t) = \sum_{n=1}^{\infty} c_n \sin(nk_1 x)\cos(n\omega_1 t + \phi_n)$

where: $k_1 = 2\pi/\lambda_1$ and: $\omega_1 = 2\pi f_1$ with: $v = f_1\lambda_1 = \omega_1 / k_1$

T = string tension

$\lambda_1 = 2L$ $\qquad v = \lambda_1 f_1 = \sqrt{\frac{T}{\mu}}$ $\qquad \mu = \frac{M}{L}$ $\qquad M$ = mass of string

$k_n = nk_1 = \frac{2\pi}{\lambda_n}$ $\qquad v = \lambda_n f_n = \omega_n / k_n$ $\qquad L$ = length of string

$\lambda_n = \lambda_1 / n$ and: $f_n = nf_1$ where: $n = 1, 2, 3, 4, ...$

Hierarchy of tones/harmonics = harmonic series;

e.g. $y(x,t) = \sum_{n=1}^{\infty} b_n \sin(nk_1 x)\cos(n\omega_1 t)$

= superposition of waves of frequencies $f_n = nf_1$ on a vibrating string

Note that $f_2 = 2f_1$ means that f_2 is one ***<u>octave</u>*** higher than f_1.

Ratio $f_2/f_1 = 2:1$ The musical interval between harmonics 1 and 2 is an ***<u>octave</u>***.
Ratio $f_3/f_2 = 3:2$ The musical ***<u>interval</u>*** between harmonics 2 and 3 is a ***<u>fifth</u>***, *etc.*