**Sound Intensity & Human Hearing:**

The human ear is exquisitely sensitive to detecting sounds (*n.b.* dogs hear ~ 100× better!) Due to this large dynamic range, our ears have an ~ *logarithmic* response to sound intensity.

Sound Intensity, $I$ (*RMS Watts*/$m^2$) $\propto \xi^2_{rms}$ (*RMS* particle displacement amplitude)$^2$
as well as: $I \propto u^2_{rms}$ (*RMS* particle velocity amplitude)$^2$
as well as: $I \propto p^2_{rms}$ (*RMS* over-pressure amplitude)$^2$

The relationship of the {magnitude of the} *RMS* sound intensity at a given point $\vec{r}$ in a sound field associated *e.g.* with monochromatic/pure-tone sine-type travelling plane waves propagating in free air (*i.e.* the great outdoors) to the above quantities is:

$$\boxed{I_{rms}(\vec{r},t) = p^2_{rms}(\vec{r},t)/\rho_o c = \rho_o c \cdot u^2_{rms}(\vec{r},t) = \rho_o c \cdot \omega^2 \xi^2_{rms}(\vec{r},t) = c w^{ac}_{rms}(\vec{r},t)}$$

where:

$p_{rms}(\vec{r},t)$ = *RMS* over-pressure amplitude (*RMS Pascals*) at the point $\vec{r}$

$\rho_o$ = equilibrium mass density of dry air (@ *NTP*) = 1.204 $kg/m^3$

$c = v_{air}$ = longitudinal propagation speed of sound in dry air = 343 $m/s$ (@ *NTP*)

$u_{rms}(\vec{r},t)$ = so-called *RMS* longitudinal particle velocity amplitude (*RMS m/s*) at the point $\vec{r}$

$\omega = 2\pi f$ = angular frequency (*radians*/*sec*), $f$ = frequency (*Hz*)

$\xi_{rms}(\vec{r},t)$ = *RMS* longitudinal particle displacement amplitude (*RMS meters*) at the point $\vec{r}$ .

The *RMS* acoustic energy *density* (*Joules*/$m^3$) at the point $\vec{r}$ is given by:

$$\boxed{w^{ac}_{rms}(\vec{r},t) = p^2_{rms}(\vec{r},t)/\rho_o c^2 = \rho_o u^2_{rms}(\vec{r},t) = \rho_o \omega^2 \xi^2_{rms}(\vec{r},t)}$$

The quantity $\boxed{z(\vec{r}) \equiv p_{rms}(\vec{r})/u_{rms}(\vec{r}) \ \ (Pa\text{-}s/m)}$ is known as the *specific acoustic impedance* of the medium, which for monochromatic plane waves propagating in a free air sound field is a constant (*i.e.* independent of frequency): $\boxed{z^{free\ air}_{ac} \equiv z_o = \rho_o c \simeq 415\, kg/m^2\text{-}sec = 415\,\Omega_{ac}}$ (@ *NTP*).

Thus, we also see that for such plane waves propagating in a free-air sound field that:

$$\boxed{I_{rms}(\vec{r},t) = p_{rms}(\vec{r},t)\cdot u_{rms}(\vec{r},t) = p^2_{rms}(\vec{r},t)/z(\vec{r}) = u^2_{rms}(\vec{r},t)\cdot z(\vec{r}) = c w^{ac}_{rms}(\vec{r},t)}.$$

Note that *RMS* quantities are frequently/often used in acoustical (and other) physics because for a pure-tone/single-frequency sine wave-type sound, they conveniently correspond to the *time-averaged* value of that quantity:

$$\langle I \rangle = \frac{1}{\tau}\int_{\substack{one\\cycle}} I(t)\,dt = \frac{1}{2} I_{peak} = I_{rms} \propto \frac{1}{\tau}\int_{\substack{one\\cycle}} p^2(t)\,dt = \frac{1}{\tau} p_o^2 \int_{\substack{one\\cycle}} \sin^2 \omega t\,dt = \frac{1}{\tau}\frac{1}{2} p_o^2 = \frac{1}{\tau} p^2_{rms}$$

$p_{rms}$ = *RMS* (*R*oot-*M*ean-*S*quare) of sound (over-)pressure amplitude (*RMS Pascals* = *RMS Newtons*/$m^2$)

For a pure-tone (*i.e* single-frequency) sine wave: $p_{rms} \equiv \frac{1}{\sqrt{2}} p_o$ {$\Rightarrow$ $p^2_{rms} \equiv \frac{1}{2} p_o^2 = \langle p_o^2 \rangle$ }