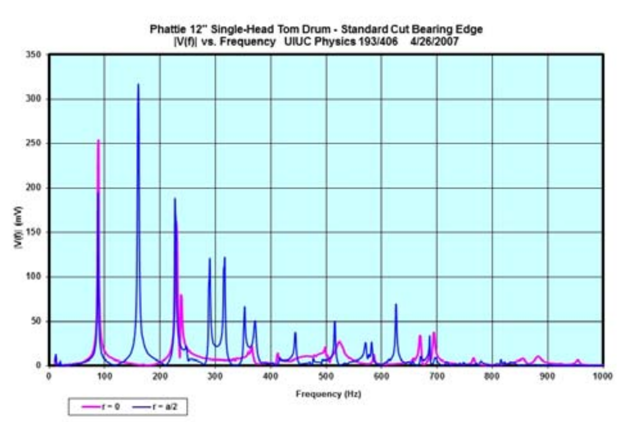
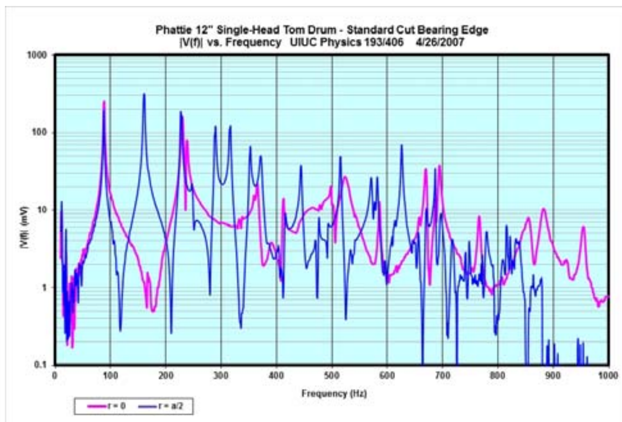
The modal frequencies of a circular membrane are $f_{m,n} = \omega_{m,n}/2\pi = vk_{m,n}/2\pi$, but we also have the relation $k_{m,n} = x_{m,n}/a$ where $x_{m,n}$ is the value of the n^{th} non-trivial zero of the m^{th}–order Bessel function $J_m(x_{m,n}) = J_m(k_{m,n}a) = 0$, *e.g.* for *m* = 0 and *n* = 1,2,3,4,5… then $J_0(x_{0,n}) = J_0(k_{0,n}a) = 0$ when $x_{0,n} = k_{0,n}a = 2.405, 5.520, 8.654, 11.793, 14.931, \ldots$ respectively.

The speed of propagation of transverse waves on a (perfectly-compliant) circular membrane clamped at its outer edge is $v = \sqrt{T_\ell/\sigma}$ where $T_\ell\,(N/m)$ is the surface tension (per unit length) of the membrane and $\sigma\,(kg/m^2)$ is the areal mass density of the membrane/drum head. Thus:

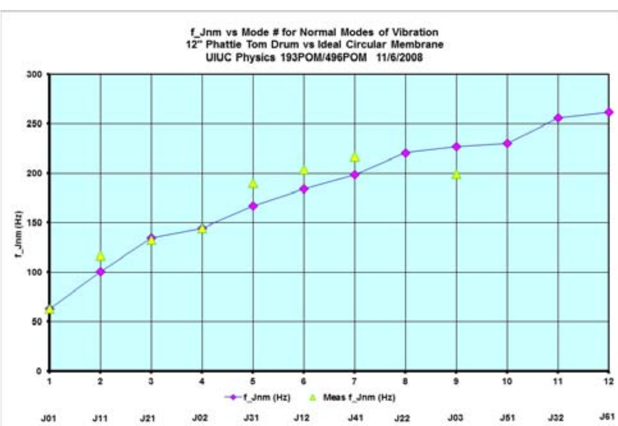
$$\boxed{f_{m,n} = \frac{\omega_{m,n}}{2\pi} = \frac{vk_{m,n}}{2\pi} = \frac{k_{m,n}}{2\pi}\sqrt{\frac{T_\ell}{\sigma}} = \frac{k_{m,n}a}{2\pi a}\sqrt{\frac{T_\ell}{\sigma}} = \frac{x_{m,n}}{2\pi a}\sqrt{\frac{T_\ell}{\sigma}}\ (Hz)}$$

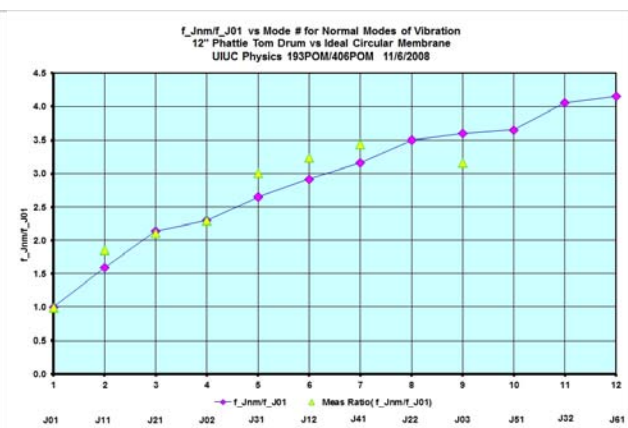
<u>Example</u>: A frequency scan of the resonances associated with the modal vibrations of a Phattie 12" single-head tom drum using the UIUC Physics 193/406POM modal vibrations PC-based data acquisition system is shown in the figures below:





<u>Data *vs.* Theory Comparison of Phattie 12" Tom Drum J_{nm} Modal Frequencies</u>:





n.b. The clear mylar drum head on the Phattie 12" tom drum does have ***<u>finite</u>*** ***<u>stiffness</u>***, *i.e.* it is ***<u>not</u>*** ***<u>perfectly</u>*** compliant, as for an ***<u>ideal</u>*** circular membrane… which affects/alters the resonance frequencies of modes of vibration of drum head….