Vibrations of Ideal Circular Membranes (*e.g.* Drums) and Circular Plates:

Solution(s) to the wave equation in 2 dimensions – this problem has cylindrical symmetry ⇒ Bessel function solutions for the radial (r) wave equation, harmonic {sine/cosine-type} solutions for the azimuthal (φ) portion of wave equation. Please see/read "Mathematical Musical Physics of Wave Equation – Part II" *p.* 16-20 for further details…

Boundary condition: Ideal circular membrane (drum head) is ***clamped*** at radius a ⇒ must have transverse displacement ***node*** at $r = a$.

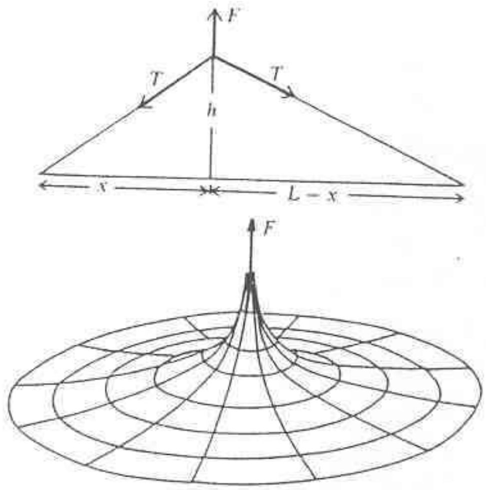

FIGURE 3.4. Reaction of a string and membrane to a force applied at a point.

The 2-D wave equation for transverse waves on a drum head – approximated as a cylindrical membrane has Bessel function solutions in the radial (r) direction and cosine-type functions in the azimuthal (φ) direction (see P406 Lect. Notes "Mathematical Musical Physics of the Wave Equation – Part II", *p.* 16-20): $\psi_{m,n}^{disp}(r,\varphi,t) = A_{m,n} J_m(k_{m,n} r)\cos(m\varphi)\cos(\omega_{m,n} t)$ where $J_m(x_{mn}) = J_m(k_{mn}r)$, $x_{mn} = k_{mn}r$ (*n.b.* dimensionless quantity), k_{mn} = wavenumber = $2\pi/\lambda_{nm}$. The integer index m = 0,1,2,3... refers both to the order # of the {ordinary} Bessel function (in the radial, r-direction) ***and*** also the azimuthal (φ-direction) node #. The index n = 1,2,3,4... refers to the n^{th} non-trivial zero of the Bessel function $J_m(x_{mn})$, *i.e.* when $x_{mn} = k_{mn}a = 0.0$. The boundary condition that the circular membrane is rigidly attached at its outer radius $r = a$ requires that

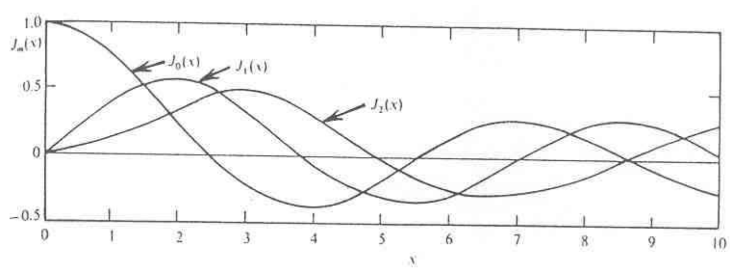

FIGURE 3.5. First three Bessel functions.

there be a transverse displacement ***node*** at $r = a$, *i.e.* $\psi_{m,n}^{disp}(r = a,\varphi,t) = 0$. This gives rise to distinct modes of vibration of the drum head (see 2-D and 3-D pix on next page):