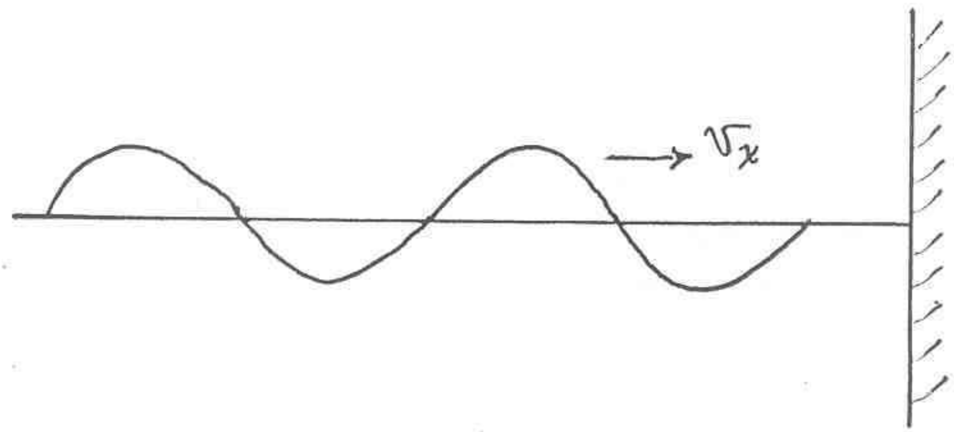
<u>Standing Waves</u>

Created when harmonic traveling wave reflects *e.g*. from a ***<u>fixed</u>*** (*i.e*. rigid, immovable) end:



x* = *L

- Resultant wave (in the $x < L$ region) is of the general form:

$$y(x,t) = f(x - vt) - f(-x - vt) \quad \text{where: } f(x - vt) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{\tau}\right)\right]$$

Analytic form for two counter-propagating traveling waves:

$$y(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{\tau}\right)\right] - A\sin\left[2\pi\left(-\frac{x}{\lambda} - \frac{t}{\tau}\right)\right] \quad \Leftarrow$$

_n.b._ the – sign for the left-moving reflected wave is due to the polarity flip (_i.e._ phase change of 180° upon reflection) of the incident right-moving wave from the fixed/immovable endpoint.

$$= A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{\tau}\right)\right] + A\sin\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{\tau}\right)\right] \quad \Leftarrow n.b. \ \boxed{\sin(-u) = -\sin u} \ \ i.e. \ \textbf{odd} \text{ fcn of } u.$$

Now use the trigonometric identity: $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$$y(x,t) = A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{\tau}\right) - \cancel{A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi t}{\tau}\right)}$$

$$+A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{\tau}\right) + \cancel{A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi t}{\tau}\right)}$$

Thus: $y(x,t) = 2A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{\tau}\right)$ for ***standing*** wave

= two ***counter-propagating traveling*** waves.

- Note: The analytic form describing the transverse displacement $y(x,t)$ associated with a **<u>standing</u>** wave is the ***<u>product</u>*** of two harmonic functions: fcn(space) × fcn(time).