The Sinc² function – Sinc²(*x*) *vs. x*, where $x = \frac{1}{2}\delta_{max} = (\pi a \sin \theta / \lambda)$, relevant for diffraction of sound (or light) through a narrow slit/aperture of lateral width *a* is shown in the figure below, as a <u>semi-log</u> plot. The global maximum of the intensity/power is in the central lobe, near $|x| \sim 0$.



Diffraction minima occur when $x = \frac{1}{2}\delta_{\max} = (\pi a \sin \theta / \lambda) = \pm \pi, \pm 2\pi, \pm 3\pi, \ldots = \pm m\pi, m = 1, 2, 3, \ldots$

Diffraction Through a Circular Aperture of Radius, R:

A more realistic situation for diffraction of sound is that of diffraction through a circular aperture. Diffraction occurs in <u>all</u> sound-generating transducers, such as loudspeakers. For a circular loudspeaker of radius R (*n.b.* also mounted on an <u>infinite</u> baffle) the angular intensity distribution $I(\theta)$ resulting from the sound diffracting from the aperture of the loudspeaker is given by:

$$I(\theta) = I_o \left[\frac{2J_1(\rho)}{\rho}\right]^2 = I_o \left[\frac{2J_1(kR\sin\theta)}{kR\sin\theta}\right]^2$$

where θ is the polar angle from the axis of the loudspeaker, $\rho \equiv kR \sin \theta$ and $J_1(\rho)$ is the ordinary Bessel function of order 1. Bessel functions frequently arise in situations where circular/cylindrical symmetry is involved. The Bessel function of order *n*, $J_n(x)$ can be expressed as a power series expansion in *x*:

$$J_{n}(x) = \frac{x^{n}}{2^{n}\Gamma(n+1)} \left\{ 1 - \frac{x^{2}}{2(2n+2)} + \frac{x^{4}}{2 \cdot 4(2n+2)(2n+4)} - \ldots \right\} = \sum_{k=0}^{\infty} \frac{(-1)^{k} (x/2)^{n+2k}}{k!\Gamma(n+k+1)}$$

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