

Interference Effects from Multiple Sound Sources – Phased Arrays:

Obviously, the phasor diagram can be extended to multiple (*i.e.* N) identical sound sources, each with sound intensity I_o arranged in a line (*e.g.* the x -axis), each separated by a lateral distance d from each other, and interfering with each other – analogous to multiple, or N -slit interference in optics!

The intensity distribution for N -slit interference on a transverse screen is given by:

$$I_{tot} (x) = I_o \left\{ \frac{\sin^2 \left(\frac{N\delta}{2} \right)}{\sin^2 \left(\frac{\delta}{2} \right)} \right\}$$

Minima – *i.e.* intensity zeroes (complete destructive interference) occur when the numerator factor $N\delta/2 = \pm\pi, \pm2\pi, \pm3\pi, \dots = n\pi, n = \pm1, \pm2, \pm3, \dots$ *except* when the denominator factor simultaneously has $\delta/2 = \pm\pi, \pm2\pi, \pm3\pi, \dots = n\pi, n = \pm1, \pm2, \pm3, \dots$ then we have a global maximum of the intensity, where $I_{tot} = N^2 I_o$.

The limiting case is where the number of sources/slits, $N \rightarrow \infty$ – *i.e.* a continuum of closely-spaced/immediately adjacent/contiguous, infinitesimally small sound sources, all in phase with each other, as in the 2-sound source case described immediately above. This limiting case describes the phenomena of diffraction of sound waves (or light waves) *e.g.* through a constricting aperture!

The phasor method can also be used for obtaining the intensity distribution associated *e.g.* with a 2-dimensional phased array of sound sources since, as in the case for light/EM waves, sound interference effects along one axis (*e.g.* x) do **not** interfere with those along a different axis (*e.g.* y). For example, the overall intensity distribution for a 2-dimensional rectangular array of N_x and N_y sound sources in the far-field limit is given by the product expression:

$$I_{tot} (x, y) = I_o \left\{ \frac{\sin^2 \left(\frac{N_x \delta_x}{2} \right)}{\sin^2 \left(\frac{\delta_x}{2} \right)} \right\} \left\{ \frac{\sin^2 \left(\frac{N_y \delta_y}{2} \right)}{\sin^2 \left(\frac{\delta_y}{2} \right)} \right\}$$

where in the far-field limit: $\delta_x = 2\pi d_x \sin \theta_x / \lambda$ and $\delta_y = 2\pi d_y \sin \theta_y / \lambda$ (in radians).

Additional info & plots on 1-D and 2-D N -slit “far-field” interference is available on the Physics 406 Software webpage at the following URL:

http://courses.physics.illinois.edu/phys406/406pom_sw.html